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Peirce and Logicism: Notes Towards an Exposition

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My damned brain has a kink
in it that prevents me from
thinking as other people think.
[C.S. Peirce][1]

1. *Introduction:*

Was Peirce a logicist? If one has to give a simple answer, certainly it must be "no;" but the issue is sufficiently far from straightforward that a simple answer is not fully adequate. How far from straightforward the issue is might be illustrated by the fact that Murphey opens chapter XII of *The Development of Peirce's Philosophy* with a discussion of Peirce's objections to the logicist position as represented by Dedekind, but closes chapter XIII with the observation that "[i]n spirit. . . Peirce has more in common with the logicistic school than with intuitionism."[2] He makes no comment about the apparent tension.

In fact, the evidence seems to be that, though staunchly opposing one characteristic logicist thesis, Peirce sympathized with another. Since the two theses appear to stand or fall together, as Frege assumed they did, this raises some intriguing questions, both exegetical and philosophical. In hopes that here, as elsewhere, there may be something important to be learned thanks to the "kink" in Peirce's brain, I offer in this paper my (preliminary, and pretty tentative) attempt to spell out something of his conception of the relation of mathematics to logic.

2. *Background: Peirce's knowledge of logicism:*

There is no reference to logicism in the indices to the *Collected Papers,*[3] nor in the indices to the first four volumes of the *Chronological Edition,*[4] nor in the indices to the *New Elements of Mathe-*

matics.[5] Only in his discussions of Dedekind, whom he mentions at 4.239 (1902) as holding that "mathematics is a branch of logic"—a thesis Peirce immediately rejects—does he come even close to an explicit discussion of logicism.

Frege recognized Dedekind as having anticipated the logicist view, writing in the *Grundgezetze* that "Herr Dedekind, like myself, is of the opinion that the theory of numbers is part of logic," but went on to say that Dedekind's work "hardly contributes to [this opinion's] confirmation" because it is insufficiently rigorous.[6] But there is no reference to Frege in either the *Collected Papers* or the *New Elements*, and the few references in the *Chronological Edition* are all to the editors' introductions, not to Peirce's text. Peirce must have known something of Frege's work; Schröder sent him a copy of his review of the *Begriffsschrifft*, and Christine Ladd listed this review and the *Begriffsschrifft* itself in the bibliography of her paper, "On the Algebra of Logic," in the *Studies in Logic* published by members of the Johns Hopkins University and edited by Peirce.[7] I conjecture that Schröder's on the whole hostile review, and especially his claim that, apparently in ignorance of Boole's work, Frege was effectively just transcribing his calculus of judgements in a clumsy new notation, may have disinclined Peirce to take any further interest in Frege's work.[8]

Peirce reviewed Russell's *Principles of Mathematics* in 1903, but the review amounts only to a cursory paragraph; Murphey conjectures that Peirce may not actually have read the book at the time he wrote the review.[9] Apart from this, all the references to Russell in the *Collected Papers* are supplied by the editors, as is the one reference in the *Chronological Edition*. The several references in the *New Elements* are all dismissive, the most notable describing Russell and Whitehead as "blunderers, continually confusing different questions" (III/2, p. 785, 1906).

But in their footnote to 3.43-4 (1876) Hartshorne and Weiss comment on the affinity between Peirce's definition of cardinals, and that of *Principia Mathematica*. In the article on Peirce in the *Dictionary of American Biography*, P.W., referring to the same 1867 paper, writes that Peirce "clearly anticipated the method for the derivation and definition of number employed in the epochal *Principia Mathematica*."[10] And the same point is made in Eisele's *Studies in the Scientific and Mathematical Philosophy of C.S. Peirce*; Peirce, Eisele writes, anticipated "many of the ideas to be found in . . . *Principia Mathematica*."[11]

In the circumstances, the best strategy seems to be to begin by considering the characteristic theses of logicism, and investigating Peirce's attitude to those theses.

3. *Two Characteristic Theses of Logicism:*

Logicism comes in two versions: a narrower, concerning the relation of *arithmetic* to logic, and a broader, concerning the relation of *mathematics* to logic. Frege's logicism was of the narrower variety, Russell and Whitehead's of the broader.[12] In either version, there are two theses central to logicism, one formal, the other epistemological. The former is to the effect that all the special concepts of mathematics [arithmetic] are definable in purely logical terms, and all the theorems of mathematics [arithmetic] are then derivable from purely logical principles. For short, this is the thesis that *mathematics [arithmetic] is reducible to logic*, to which I shall refer as (L1). Closely associated are the theses that the propositions of mathematics [arithmetic] are analytic, and (in Frege at least) that mathematical [arithmetical] objects are abstract, neither mental nor physical. The epistemological thesis of logicism is to the effect that because of the certainty or self-evidence of the logical axioms, the reducibility of mathematics [arithmetic] to logic explains the peculiar security, the *a priori* character, of mathematical [arithmetical] knowledge. For short, this is the thesis that *the epistemological foundations of mathematics [arithmetic] lie in logic*, to which I shall refer as (L2).

Frege took it for granted that (L1) and (L2) stand or fall together:

> . . . arithmetic is a branch of logic and need not borrow any ground of proof whatever from experience or intuition. . . . Every axiom which is needed must be discovered . . . it is just the hypotheses which are made without clear consciousness that hinder our insight into the epistemological nature of a law.[13]

And so, it seems, until he felt the need to hedge his bets about the rationale for the Axiom of Infinity, did Russell:

> The connection of mathematics with logic . . . is exceedingly close. The fact that all mathematical constants are logical constants, and that all the premisses of mathematics are concerned with these gives, I believe, the precise statement of what philosophers have meant in asserting that mathematics is *à priori*.[14]

But the textual evidence seems to indicate that Peirce sympathized with something like (L1) while resolutely opposing anything like (L2).

4. *Peirce's sympathy with the first logicist thesis:*

In the second Lowell lecture of 1866 (*CE* 1, p. 386), Peirce writes that "mathematical demonstration *can* be reduced to syllogism;" by 1867 he is claiming much more than this, opening a paper entitled "Upon the Logic of Mathematics" thus:

> The object of this paper is to show that there are certain general propositions from which the truths of mathematics follow syllogistically, and that these propositions may be taken as definitions of the objects under the consideration of the mathematician . . . (3.20; *CE* 2, pp. 59-60)

What follows appears to be an attempted reduction of arithmetical propositions to Boolean logic. And in the course of the paper Peirce presents a discussion of cardinal number which, as Hartshorne and Weiss, P.W., and Eisele all remark, bears a striking affinity with the definition offered, much later, in *Principia Mathematica*:

> . . . let the letters, in the particular application of Boole's calculus now supposed, be terms of second intention which relate exclusively to the extension of first intentions. Let the differences of the characteristics of things and events be disregarded, and let the letters signify only the differences of classes as wider or narrower. . . . Thus *n* in another case of Boole's calculus might, for example, denote "New England States;" but in the case now supposed, all the characters which make those States what they are being neglected, it would signify only what essentially belongs to a class which has the same relations to higher and lower classes which the class of New England States has,—that is, a collection of *six*.
>
> In this case, the sign of identity will receive a special meaning. For, if *m* denotes what essentially belongs to a class of the rank of "sides of cube," then $n \underset{,}{=} m$ will imply, not that every New England State is a side of a cube, and conversely, but that what ever essentially belongs to a class of the numerical rank of "New England States" essentially belongs to a class of the numerical rank "sides of a cube," and conversely. *Identity* of this sort may be termed *equality*. . . . (3.43-4; *CE*, 2, pp. 68-9)

Peirce's first paper on the logic of relatives appeared in 1870. Subsequently, in a paper in which he returns to the relation of arithmetic to logic, and which opens:

> Nobody can doubt the elementary propositions concerning number. . . The object of this paper is to show that they are strictly syllogistic consequences from a few primary propositions . . . which I here regard as definitions . . . (3.252; *CE*, 4, p. 299, 1881)

he observes that these "syllogistic" derivations will require the logic of relatives. It was of course also crucial to Frege's program that by developing a logic in which relations are expressible he overcame the difficulty which frustrated Leibniz's proto-logicist project, that arithmetic cannot be reduced to Aristotelian syllogistic logic because of the inability of that logic to represent relations.

At 4.88 (1893) Peirce points out how numerical propositions of the form "There are *n Fs*" may be "syllogistic conclusions from particular propositions;" for example, that from "Some A is B" and "Some not-A is B" it follows that there are at least two Bs. He continues (4.93) by remarking that "there are various ways in which arithmetic may be conceived to connect itself with and spring out of logic"—the way just indicated, and the ways described in the two earlier papers discussed above.

A discussion somewhat misleadingly entitled "Synthetical Propositions A Priori" (*NEM*, IV, pp. 82ff., 1892) insists that mathematical propositions either "define an ideal hypothesis (in the mathematical sense)" or else are "deductions from those definitions" (p. 82). To illustrate this, Peirce continues, he will "prove from definitions that 7 + 5 = 12." "Only an ignorance of the logic of relatives," he continues, "has made another opinion possible."

It certainly looks, in short, as if Peirce sympathized with something much like (L1).

The position with respect to the thesis, which for Frege and Russell is virtually identified with the logicist thesis I have called

(L1), that the truths of mathematics [arithmetic] are analytic, is a bit more ambiguous. In the paper of 1893 mentioned earlier, Peirce remarks that the way arithmetic springs out of logic "is sufficient to refute Kant's doctrine that the propositions of arithmetic are 'synthetical'" (4.91), and goes on to criticize Mill's conception of arithmetical laws as very general empirical propositions. He is keen, however, to insist that "those who [like myself] maintain that arithmetical truths are logically necessary" are not "*eo ipso* saying that they are verbal in their nature." Similarly, in the paper of 1892 in which Peirce offers his proof of "7 + 5 = 12" from definitions, he concludes that "the proposition in question is analytical or explicatory," but makes a point of adding, "but, no doubt, Kant had a very narrow conception of explicatory propositions, owing to his knowing nothing of the logic of relatives" (*NEM*, IV, p. 84). A discussion of "essential predication" from 1901 makes it clear that Peirce is uneasy about the way Kant's definition suggests that what is analytically true must be obvious: an essential predication is one where the predicate is contained in the essence of the subject, hence, analytic in Kant's sense; but, Peirce continues, neither Kant nor the scholastics realized that "an indefinitely complicated proposition, very far from obvious, may . . . be deduced . . . by the logic of relatives, from a definition of the utmost simplicity . . . ; and this may contain many notions not implicit in the definition" (2.361, 1901). In view of this, it is not so surprising that, by 1902, one finds Peirce ***denying*** that mathematical truths are analytic; of Kant's conception of mathematical truths as synthetic ***a priori***, he remarks that it is true, at any rate, that "they are not, for the most part, what he called analytical judgements; . . . the predicate is not, in the sense intended, contained in the definition of the subject" (4.232). I think there is no real contradiction here, only a verbal shift; Peirce holds, on the one hand, that mathematical truths are deducible from definitions, but insists, on the other, that this does not mean that they are trivial, obvious, or merely verbal.[15]

But it should already be apparent that Peirce should not be expected to take (L1) to have the epistemological consequences which Frege and Russell supposed; for he is chafing against the distinction of analytic and synthetic. Mathematical truths straddle the usual distinction: they are not empirical generalizations, but necessary truths—necessary truths, however, discoverable by observation of or experimentation on imagined diagrams.

The position with respect to the other thesis closely associated, in Frege at least, with (L1), mathematical Platonism, is less straightforward yet.[16] True, at 4.118 (1893) Peirce speaks of "the Platonic world of pure forms with which mathematics is always dealing," and in the prospectus for his 12-volume *Principles of Philosophy* (c.1893) he entitles the third volume, *Plato's World: an Elucidation of the Ideas of Modern Mathematics*.[17] But at 4.161 (c.1897) we read: "[the system of abstract numbers] is a cluster of ideas of individual things, but it is not a cluster of real things. It . . . belongs to the world of ideas, or Inner World. But nor does the mathematician, though he "creates the idea *for himself*, create it absolutely." This is pretty baffling until one reads, at 6.455 (1908), "[o]f the three Universes of Experience . . . the first comprises all mere Ideas, those airy nothings to which the mind of poet [or] pure mathematician . . . *might* give local habitation . . ."[18] The idea seems to be that the constructions of the mathematician actualize what already had the status of possibility, of firstness. So while in the earlier piece Peirce denied that numbers are real, i.e., independent of thought, he now writes that "the fact that their Being consists in mere capability of getting thought, not in anybody's Actually thinking them, saves their Reality." If this is Platonism, it is Platonism of a very distinctively Peircean stripe.[19]

5. *Peirce's repudiation of the second logicist thesis:*

Peirce's repudiation of (L2) seems almost completely unambiguous. I say "almost" because of the following passage, the only one I have found that even appears to suggest any sympathy with (L2); it comes from chapter 7 of a ms "toward a logic book" of 1872-3, entitled "Of Logic as a Study of Signs:"

> The business of Algebra in its most general signification is to exhibit the manner of tracing the consequences of supposing that certain signs are subject to certain laws. And it is therefore to be regarded as part of Logic. (*CE*, 3, p. 83)

What immediately follows, however, is an argument against "certain logicians of some popular repute" (the editors remark, "the reference is almost certainly to W. Stanley Jevons") who claim that algebra is "inapplicable to logic." In view of this, and of the fact that a little later (p. 92) Peirce is found explaining how one bit of algebraic notation, "a—<b," may be interpreted as representing "a is smaller than b" or "all a is b" or "b is a consequence of a," the most plausible explanation of the passage quoted seems to be as insisting on the usefulness of algebraic notation to logic, not as claiming the epistemic dependence of algebra on logic.[20]

Every other relevant text I have come across seems to indicate unambiguously that Peirce was strenuously opposed to the thesis that mathematics is founded epistemologically on logic.[21]

For Peirce, it is Dedekind who represents the idea that mathematics is a branch of logic. Apparently Peirce's father was, at the time he was writing his *Linear Associative Algebra*, attracted to something like this view; Peirce reports that he did his best to dissuade him.[22] And the other evidence of Peirce's repudiation of anything like (L2) is overwhelming. "We homely thinkers believe that . . . the safest way is to appeal for our logical principles to the science of mathematics" (3.427, 1896). "It does not seem to me that mathematics depends in any way upon logic" (4.228, 1902). "[L]ogic depends on mathematics" (4.240, 1902). "Mathematics is not subject to logic. Logic depends on mathematics" (2.191, 1902). "Logic can be of no avail to mathematics; but mathematics lays the foundations on which logic builds" (2.197, 1902). [M]athematics . . . has no need of any appeal to logic" (4.242, 1902). "[T]rue mathematical reasoning is so much more evident than it is possible to render any doctrine of logic proper . . . that an appeal in mathematics to logic would only embroil a situation" (4.243, 1902). "[T]here is no more satisfactory way of assuring ourselves of anything than the mathematical way of assuring ourselves of mathematical theorems. No aid from logic is called for in this field" (2.192, 1902). "*[M]athematics is almost, if not quite, the only science which stands in no need of aid from the science of logic*" (2.81, 1902). "[T]here are but five theoretical sciences which do not more or less depend on the science of logic . . . the first is mathematics . . . Mathematics has no occasion to inquire into the theory of validity of its own arguments; for these are more evident than any such theory could be" (2.120, 1902).

6. *The explanation of Peirce's apparent sympathy with (L1) and repudiation of (L2) does not lie in the distinction between broad and narrow logicism:*

On the assumption that (L1) and (L2) stand or fall together, Peirce's position stands in need of explanation. One diagnosis that suggests itself, since the passages which indicate sympathy with something like (L1) seem to be concerned with the reducibility of *arithmetic* to logic, while the passages repudiating (L2) seem to be concerned with the epistemic priority of *mathematics* over logic, is that Peirce sympathizes with (L1) in the narrow interpretation ("arithmetic is reducible to logic") but rejects (L2) in the broad interpretation ("the epistemic foundations of mathematics lie in logic"). There are a couple of complicating factors: Peirce dislikes the traditional division of mathematics into algebra and geometry: (1.283, 4.247, both 1903), and he sometimes uses "geometry," *simpliciter*, to refer to physical geometry (3.427, 1896); nevertheless, this explanation can be decisively ruled out.

Peirce's sympathy with (L1) *does* include its application to pure geometry. This is indicated by his observation at 3.526 (1897) that "for projective geometry, Schubert has developed an algebraical calculus which has a most remarkable affinity to the Boolian logic;" and cf. 4.131 (1893) which describes Schubert's *Calculus of Enumerative Geometry* as "the most extensive application of the Boolian algebra which has ever been made . . . the classical treatise upon geometry as viewed from the standpoint of arithmetic."[23] But most decisive is this passage from the paper entitled "Synthetical Propositions A Priori," already cited above; where, after his proof of "7 + 5 = 12" from definitions, and his comment that this judgement is analytic (though not in quite Kant's narrow sense), Peirce continues:

> Some have been of the opinion that while arithmetical propositions are analytic, geometrical ones are synthetic. But I am certain they are all of the same character. (*NEM*, IV, p. 84, 1892)

7. *The explanation of Peirce's apparent sympathy with (L1) and repudiation of (L2) does not lie in a simple change of mind:*

Another diagnosis that suggests itself, in part because the passages I have cited that seem to bespeak sympathy with (L1) are generally earlier than those I have cited as indicating Peirce's repudiation of (L2), is that Peirce may have shifted from early logicist sympathies to a later disenchantment with this kind of approach.

This conjecture might be thought to be supported by the fact that Peirce many years later described the paper of 1867 in which he claimed to show that arithmetical propositions are derivable syllogistically from definitions as, he trusted, "by far the worst I have ever written" (4.333, c.1905). But this is not decisive: for he continues by remarking that the paper was "founded on an interesting idea, worthy of a better development." (The context suggests that he thought his account of cardinals in that paper, the one so close to *Principia Mathematica*, could be improved; but it also indicates that by this time—1905—Peirce was firmly of the opinion that ordinals are primary, not cardinals.)[24] In any case, for all his harshness about the 1867 paper, Peirce subsequently described the paper of 1893 which I have also cited as indicating sympathy with (L1) as the *strongest* he had ever written.[25]

Another passage which might at first blush be thought to suggest a change of mind, where Peirce remarks that the "nearest approach to a logical analysis of mathematical reasoning" was Schröder's statement of Dedekind's analysis in a logical algebra devised by Peirce himself, but that "the soul of the reasoning has even here not been caught in the logical net" (4.426, c. 1903), turns out, a few paragraphs later, to be only paving the way for the observation that the system of existential graphs, being diagrammatic, represents mathematical reasoning better than any algebraic notation (4.429).[26]

Not only is there no decisive evidence of a change of mind; there is also pretty decisive evidence against it. At 4.90 (1893) Peirce expresses *both* sympathy with something like (L1) ***and*** in the same sentence antipathy to anything like (L2): ". . . the whole theory of numbers belongs to logic; or rather, it would do so were it not, as mathematics, *prelogical*, that is, even more abstract than logic." And then there is also Peirce's reminiscence of trying to persuade his father against "the opinion that Dedekind long afterward embraced" (that mathematics is part of logic), which must refer to a time before the publication of Benjamin

Peirce's *Linear Associative Algebra* in 1870—which certainly seems to rule out treating Peirce's repudiation of (L2) as a late development.

8. *The explanation lies, at least in part, in an ambiguity in Peirce's use of "logic:"*

When Peirce remarks in the paper of 1893 (4.85ff.) discussed above that "there are several ways in which arithmetic may be conceived to spring out of logic," "logic" evidently means "formal deductive logic." But unlike Frege, who seems virtually always to mean "formal deductive logic" when he writes of "logic," Peirce uses "logic" in a whole range of ways, of which this is one of the narrowest. My present concern is not with Peirce's shift from an earlier conception of logic as a small part of semeiotic, the part dealing with the truth and falsity of sentences, to his later identification of logic and semeiotic.[27] It is rather to point out that "logic," for Peirce, often has the broad sense of "theory of reasoning" (see e.g., 4.242, 1902); that *deductive* logic is only part of logic thus broadly conceived—the branch concerned with the theory of necessary reasoning; and that Peirce holds that *formal* logic is a branch of mathematics. At 4.228 (1902), for example, he writes that "all formal logic is merely mathematics applied to logic;" and at 4.240 (1902) that "[t]here is a mathematical logic, just as there is a mathematical optics . . . Mathematical logic is formal logic," but "*[f]ormal logic is by no means the whole of logic, or even the principal part. It is hardly to be reckoned a part of logic proper*" (my italics). And sure enough, in the "Outline Classification of the Sciences" of 1903 (1.180ff.) Peirce definitely excludes "the mathematics of logic" from logic proper; the latter is classified as one of three normative sciences, the former as one of the sciences of discovery.

Let "LOGIC" mean "theory of reasoning" and "*logic*" mean "mathematical formalization of necessary reasoning." The evidence considered thus far might now be reconstrued as indicating that Peirce holds that mathematics is reducible to *logic*, but denies that mathematics is epistemically subordinate to LOGIC. If *logic* is conceived as a branch of mathematics rather than as a branch of LOGIC, the appearance of tension can be banished. Certainly the distinction seems helpful when applied to a characteristic passage like this one:

> If there is any part of logic [LOGIC] of which mathematics stands in need, it can only be that very part of logic [*logic*] which consists merely in an application of mathematics, so that the appeal will be, not of mathematics to a prior science of logic [LOGIC], but of mathematics to mathematics [*logic*]. (1.247, 1902)

I conjecture, also, that the mature Peirce may tend increasingly to prefer to use "logic" in the broadest sense, and to regard formal deductive logic at most as only a small part of it, and eventually as not part of it at all; and that he tends, understandably, therefore, more and more to downplay the importance of the reducibility of mathematics to *logic*, and more and more to stress the importance of the epistemic independence of mathematics from LOGIC. This would explain why the passages I found indicating sympathy with, as I put it, "something like (L1)" are mostly early, and those indicating antipathy to "anything like (L2)" mostly later.

But we are not yet quite out of the woods. If my diagnosis is correct, what Peirce denies when he denies that the epistemic foundations of mathematics lie in logic (which on my interpretation means, "in LOGIC") is not after all what Frege or Russell would mean by the thesis that the foundations of mathematics lie in logic—for no such distinction as that between LOGIC and *logic* is appropriate in their case. But this obviously doesn't mean that there is no real disagreement here;[28] what it means is that the real disagreement would be more perspicuously represented as follows: Peirce, like the logicists, sympathizes with the idea that mathematics is reducible to a formal deductive system, *logic*, which, however, they regard as distinct from and epistemically prior to mathematics, but which he does not; for Peirce's view is that mathematics *requires* no foundation, that it is epistemically more secure than anything that supposedly grounded it could be. Peirce's classification of *logic* as mathematics rather than LOGIC, to put it another way, can be seen as an expression of his conviction of the epistemic autonomy of mathematics. An examination of Peirce's reasons for that conviction supplies further motivation for this way of looking at it.

9. *Peirce's reasons for insisting on the epistemic autonomy of mathematics*:

After reporting how he argued with his father against the idea that mathematics is a branch of logic [LOGIC], Peirce continues by observing that "no two things could be more directly opposite than the cast of mind of the mathematician and that of the logician . . . [T]he mathematician's interest in reasoning is as a means of solving problems . . . [T]he logician . . . is interested in picking a method to pieces and finding out what its essential ingredients are," a thought echoed at 4.533 (1906). Shrewd as these remarks are, however, they are insufficient to establish the epistemic independence of mathematics from LOGIC. Indeed, Frege—who himself, though professionally a mathematician, seems to have had the logician's temperament *par excellence*—says much the same: "[m]athematicians generally are indeed only concerned with the content of a proposition and the fact that it is to be proved. What is new in this book is . . . the way in which the proof is carried out and the foundations on which it rests . . . [an] essentially different viewpoint . . ."[29]

"The difference between the two sciences is far more than that between two points of view," Peirce writes at 4.240 (1902); it is a matter of the classification of the sciences (4.134, 1891). Actually, he says a "mere" matter of the classification of the sciences, but the "mere" here seems excessively self-deprecatory in view of the importance Peirce always attached to this classification. In all the several classifications of the sciences that Peirce devised, revised and re-revised throughout his life, it seems, mathematics is at the head, and logic occupies a subordinate position.[30]

The business of logic (i.e., LOGIC), according to Peirce, is "analysis and theory of reasoning, but not the practice of it" (4.134, 1891). The connection with the epistemic priority of mathematics over LOGIC is clear when Peirce observes, in the context of a discussion of Dedekind, that mathematics is the science *which draws necessary conclusions*, LOGIC the science *of drawing necessary conclusions* (2.249, 1902). And "just as it is not necessary, in order to talk, to understand the theory of the formation of vowel sounds, so it is not necessary, in order to reason, to be in possession of the theory of reasoning"—indeed, Peirce continues, if it were, "the science of logic could never be developed" (4.242, 1902).

Peirce insists on the epistemic priority of mathematics: "the safest way is to appeal for our logical principles to the science of mathematics, where error can only go unexploded on condition of its not being suspected" (3.427, 1896); "mathematics performs its reasonings by a *logica utens* it develops for itself, and has no need for any appeal to a *logica docens*" (1.417, c.1896); "if the mathematician ever hesitates or errs in his reasoning, logic cannot come to his aid. He would be far more liable to commit similar as well as other errors there" (4.228, 1902); "mathematics is the one [*sic*] science to which . . . logic is not pertinent; for nothing can be more evident than its own unaided reasonings" (7.524, undated). Notice how Peirce uses "evident" as a matter of degree; there is no suggestion that the truths of mathematics are self-evident. The reasoning of mathematics is fallible, Peirce holds, but "there is no more satisfactory way of assuring ourselves of anything than the mathematical way . . ." (2.192, 1902).

It is surprisingly difficult to figure out whether, when Peirce says that mathematics is the science which draws necessary conclusions, he means (1) conclusions which follow necessarily from their premisses, or (2) conclusions themselves necessary, or (3) both. The symmetry of Peirce's remarks about the business respectively of mathematics and of LOGIC seems to call for the first answer, and so might his observation that mathematical truth is "hypothetical"—until one notices that he equates "hypothetical" with "non-factual" (4.232, 1902), which suggests the third answer. This (the third interpretation) would not require one to attribute to Peirce the idea that all deductively valid reasoning is mathematics,[31] and it accomodates his observation that, though LOGIC is, mathematics isn't a "positive" science (7.524, undated), and his claim that "[mathematical] necessity must spring from some truth so broad as to hold not only for the universe we know but for any world that poet could create" (1.417, c.1896).

At any rate, it is clear that Peirce conceives of mathematics as concerned with abstract structural hypotheses, its truths as applying to all possible situations with a certain structure. And here lies an explanation of his belief in the epistemic autonomy of mathematics. The abstract structures about which mathematicians reason and on which they experiment are patterns which they themselves construct, abstract, or, perhaps best, actualize; and this is why Peirce holds that mathematical reasoning, though fallible, is as secure as any reasoning could be. "[M]athematics does not relate to any matter of fact, but merely to whether one supposition excludes another. Since we . . . create the suppositions, we are competent to answer . . ." (2.191, 1902); "[in mathematical reasoning] all pertinent facts would be within the beck and call of the imagination; and . . . nothing but the operation of thought would be necessary to render the true answer" (4.232, 1902). Mathematicians may reason carelessly; but, though mathematics is, therefore, fallible, (4.233, 1902), no appeal to LOGIC could improve its security.

10. *Envoi*:

After Russell's paradox, Gödel's incompleteness theorem, the proliferation of rival set-theories, the claim that set-theory is "pure logic" and "self-evident" is no longer plausible, and the epistemological promises of the logicist program sound hollow. Here is Quine's verdict on logicism: ". . . mathematics reduces only to set theory and not to logic proper . . . the axioms of set theory have less obviousness and certainty to recommend them than do most of the mathematical theorems we would derive from them. Moreover, we know from Gödel's work that no consistent axiom system can cover mathematics even when we renounce self-evidence. Reduction in the foundations of mathematics remains mathematically and philosophically fascinating, but it does not do what the epistemologist would like of it; it does not reveal the ground of mathematical knowledge . . ."[32] This sounds remarkably, does it not, like conceding that something like (L1) is true, but denying that anything like (L2) is defensible? Here, as so often, one might say, Peirce sounds ahead of his time.

As historians of logic remind us, Peirce belongs to another tradition than the Frege-Russell-Whitehead line that came to predominate.[33] What I have offered here, though very far from a full account of Peirce's understanding of the relation of mathematics to logic, and further yet from a serious attempt to figure out what might be defensible in that account, is enough vividly to illustrate how Peirce's conceptions run obliquely to now-familiar dichotomies. Are mathematical truths analytic or synthetic? Peirce surprises us by replying: they are not descriptions of empirical fact, but neither are they merely verbal, nor obvious. Are mathematical objects created or discovered by us? Peirce surprises us by replying: "the fact that their Being consists in mere capability of getting thought, not in anybody's Actually thinking them, saves their Reality." Does mathematical knowledge depend on experience? Peirce surprises us by replying: mathematical knowledge, like all knowledge, is acquired by experience, but by ***inner*** experience, by observation of and experimentation on imagined icons. Is mathematical knowledge certain? Peirce surprises us by replying: it is fallible, because we may blunder in our reasonings, but it stands in no need of extra-mathematical warrant. Peirce's epistemology of mathematics, neither logicist nor intuitionist,[34] resistant to contemporary categories, could prove to be a valuable resource.

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NOTES

*An abridged version of this paper was read at the conference of the Society for the Advancement of American Philosophy, University of California, Santa Cruz, March 1991. The paper had its origin in a discussion at the conference of the Society at Buffalo in March 1990, where Sleeper raised the question whether Peirce was a logicist, and Houser replied that he didn't see how anyone could suppose that he was; I went away to relocate the passages which, I thought, would show that Houser was unambiguously in the right, but what I found convinced me that matters are, after all, more complex than I originally supposed. I wish to thank the many correspondents who made helpful comments on earlier drafts of this paper: Claudine Engel-Tiercelin, Luciano Floridi, Angus Kerr-Lawson, Kenneth Laine Ketner, Mark Migotti, Sidney Ratner, Richard Robin, Ralph Sleeper and, especially, Stephen Levy and Nathan Houser.

1. Attributed to Peirce in Bell, E.T., *The Development of*

Mathematics, McGraw Hill, New York and London, first edition, third impression, 1940, p. 519. I owe the reference to Houser, "Peirce as Logician," p. 7 of his typescript.

2. Murphey, Murray G., *The Development of Peirce's Philosophy*, Harvard University Press, Cambridge, MA and London, 1961; see pp. 229-30 and 287-8.

3. Peirce, C.S., *Collected Papers*, eds. Hartshorne, C., Weiss, P., and Burks, A., Harvard University Press, Cambridge, MA, 1931-58 (references by volume and paragraph number).

4. Peirce, C.S., *Writings: A Chronological Edition*, eds. Fisch, M., Kloesel, C.J.W., Moore, E.C., Roberts, D.D., Ziegler, L.A., Atkinson, N.A., Indiana University Press, Bloomington, IN, 1982— (references given as "*CE*" by volume and page number).

5. Peirce, C.S., *The New Elements of Mathematics*, ed. Eisele, C., Mouton, the Hague and Paris/Humanities Press, Atlantic Highlands, NJ, 1976 (references given as "*NEM*" by volume and page number).

6. Frege, G., *Grundgezetze der Arithmetik* (1893); English translation by Montgomery Furth, *The Basic Laws of Arithmetic*, University of California Press, Berkeley and Los Angeles, 1964, p. 4.

7. See Fisch, M., "Peirce and Leibniz" (1972), in *Peirce, Semeiotic and Pragmatism*, eds. Ketner, K.L. and Kloesel, C.J.W., Indiana University Press, Bloomington, IN, 1986, pp. 251-2 and 259 n.8.

8. "With regard to its major content, the 'conceptual notation' could be considered actually a *transcription* of the Boolean formula language. With regard to its form, though, the former is different beyond recognition—and not to its advantage. As I have said already it was without doubt developed independently—all too independently."—from Schröder's review (1880) of Frege's *Begriffsschrift*, in Bynum, T. Ward, ed., *Conceptual Notation and Related Articles*, Clarendon Press, Oxford, 1972, p. 221. Frege himself, of course, held his system to be superior to Boole's, stressing especially the ambiguity of Boole's symbolism; see his "On the Aim of the 'Conceptual Notation'," (1882), in Bynum, pp. 90-100, and "Boole's Logical Calculus and the Concept-Script" and

"Boole's Logical Formula-Language and my Concept-Script" in *Posthumous Writings*, eds. Hermes, H., Kambartel, F., Klaubach, F., trans. Long, P. and White, R., University of Chicago Press, Chicago/ Blackwell's, Oxford, 1979, pp. 9-46 and 47-52.

9. 8.171; Murphey, *The Development of Peirce's Philosophy*, p. 241. Houser informs me that Peirce returned to the *Principles* later, but the paper he began on it in 1912 was never finished.

10. Dumas Malone, ed., *Dictionary of American Biography*, Charles Scribner's Sons, New York, 1934, volume 14, p. 400.

11. Eisele, C., *Studies in the Scientific and Mathematical Philosophy of Charles S. Peirce*, ed. Martin, R.M., Mouton, the Hague, Paris, New York, 1979, p. 12.

12. ". . . the axioms of geometry are independent of . . . the primitive laws of logic, and consequently are synthetic," Frege, G., *Die Grundlagen der Arithmetik* (1884), English translation by Austin, J.L., Blackwell's, Oxford, second edition, 1974, p. 21e. "All traditional pure mathematics, including analytical geometry, may be regarded as consisting wholly of propositions about the natural numbers," Russell, B., *Introduction to Mathematical Philosophy*, (1919), reprinted in Putnam, H. and Benacerraf, P., eds, *Philosophy of Mathematics: Selected Readings*, Prentice Hall, Englewood Cliffs, NJ, first edition, 1964, p. 115.

13. Frege, *Grundgezetze,* trans. Furth, p. 29. Cf. the following remarks from the *Grundlagen*, trans. Austin: "I hope I may claim in the present work to have made it probable that the laws of arithmetic are analytic judgements and consequently a priori. Arithmetic thus becomes simply a development of logic, and every proposition of arithmetic a law of logic, albeit a derivative one" (p. 99e); ". . . it emerged as a very probable conclusion that the truths of arithmetic are analytic and a priori" (p. 118e). Dummett suggests (*Frege: Philosophy of Language*, Duckworth, London, 1973, p. xv) that Frege's work is of central importance to contemporary philosophy because it shifts the focus from epistemology to logic and philosophy of language; this, in view of the epistemological motivation for Frege's logicist program (which Dummett himself virtually

acknowledges on p. xix) is seriously misleading.

14. Russell, B., *The Principles of Mathematics*, (1903), second edition, W.W. Norton, New York, 1938, p. 8.

15. Cf. Levy, S., "Peirce's Theoremic/Collorarial Distinction and the Interconnections Between Mathematics and Logic," forthcoming, for an elegant conjecture connecting three senses in which, according to Levy, Peirce uses "analytic," with the distinction in his title.

16. Frege is a platonist all right, but Russell's position is not so straightforward. While in the *Principles of Mathematics* (1903) he maintained a realist account of classes, by the time of "Mathematical Logic as Based on the Theory of Types" ((1908), reprinted in *Logic and Knowledge*, ed. Marsh, R.C., Allan and Unwin, London, 1956) he was maintaining the "no class" theory according to which classes are deemed to be logical fictions. Cf. Quine, W.V., "Russell's Ontological Development" (1966), reprinted in *Theories and Things*, Harvard University Press, Cambridge, MA and London, 1981, 73-85.

17. Murphey, *The Development of Peirce's Philosophy*, pp. 238-9.

18. Cf. *NEM* IV, p. 268, c. 1895, for another comparison of the mathematician and the poet.

19. So I would prefer not to describe Peirce's position, as Kerr-Lawson does, as "weak Platonism," though I agree with him that Peirce would assign mathematical objects to a different category than regular existents. See his "Benacerraf's Problem and Weak Mathematical Platonism" and "Peirce's Pre-Logicistic Account of Mathematics," both forthcoming.

20. It is worthy of note that Peirce primes, double primes and triple primes the "—<"; evidently he is well aware of the difficulty potentially caused by an ambiguous algebraic symbolism, a difficulty Frege regarded as disastrous for the Boolean approach.

21. But cf. Levy, "Peirce's Theoremic/Collorarial Distinction and the Interconnections Between Mathematics and Logic;" he holds that Peirce's position on the epistemological thesis remained incon-

sistent, since he was aware that the activities of the mathematician include, e.g,. the devising of hypotheses, which comes within logic [LOGIC] in Peirce's conception.

22. Murphey, *The Development of Peirce's Philosophy*, pp. 229-30.

23. The editors give a reference to Schubert, Hermann, *Kalkul der Abzahlenden Geometrie*, Leibzig, 1897.

24. Peirce alludes to a definition of ordinals "which was substantially given by me in 1883;" the editors suggest that this may be a slip, that Peirce refers to 3.260ff., 1881. I wonder, however, whether the reference might not be to the paper of 1893, 4.85ff.. See also fn. 25 below, and cf. Levy, S., "Peirce's Ordinal Conception of Number," *Transactions of the Charles S. Peirce Society*, XXII.6, 1986, 23-42, especially his fn. 10.

25. Editors' note to 4.85 (1893) refers to "vol. 9, letters to Judge Russell," as the source of this claim. I have been unable to find the remark in the letters to Russell published in *NEM*, so cannot supply a date. Can anybody help here?

26. It may be asked whether Peirce mightn't have given up his sympathy with (L1) on learning of Russell's paradox. According to Murphey, however, there is only one reference to the paradox in all of Peirce's writings, and this is so late (1910) that a change of mind at this point would not supply the explanation we are seeking.

27. This has been well documented by the editors of the *Chronological Edition*: see *CE*, 1, pp. xxii-xxiv, xxxii-xxxv; see also, of course, Fisch, *Peirce, Semeiotic and Pragmatism*, pp. 306, 319, 320, 323-4, 326, 338-41, 343, 350, 390-1, 396, 435-6. All I have to add is the observation that as early as 1873 there is a trace of the broader conception in the title of a piece already referred to, "On Logic as the Study of Signs," *CE*, 3, pp. 82-4.

28. This point is of more general interest, since it is often taken for granted that, if a term has a different meaning in each of two theories in which it occurs, the theories cannot be genuine rivals. Cf. the

section on "meaning-variance" in my "'Realism'," *Synthese*, 73, 1987, 272-99 (but please ignore the discussion of realism *vs* nominalism earlier in this piece, which is mistaken).

29. Frege, *Grundgezetze*, trans. Furth, p. 5.

30. I rely on Kent, B., *Charles S. Peirce: Logic and the Classification of the Sciences*, McGill-Queen's University Press, Kingston and Montreal, 1987, chapter IV.

31. Cf. 7.524, n.d.: "Pure deductive logic, *insofar as it is restricted to mathematical hypotheses*, is, indeed, mere mathematics" (my italics).

32. Quine, W.V., "Epistemology Naturalized," in *Ontological Relativity and Other Essays*, Columbia University Press, New York, 1969, p. 70.

33. See, for example, Putnam, H., "Peirce as Logician," *Historia Mathematicae*, 9, 1982, 290-301; Grattan-Guiness, I., "Bertrand Russell (1872-1970) After Twenty Years," *Notes Rec. R. Soc. Lond.*, 44, 1990, 2180-306, section 8.

34. Intuitionists, of course, like Peirce, insist that mathematics is not epistemically dependent on logic, but rather the reverse. But Peirce does not, as they do, pose any challenge to the legitimacy of the non-constructive parts of classical mathematics. And though Peirce envisaged the possibility of a non-bivalent logic, his reasons are quite different from the Intuitionist. See Fisch and Turquette, "Peirce's Triadic Logic" (1966), in *Peirce, Semeiotic and Pragmatism*, 171-83. Murphey was, I should note, quite correct in seeing Peirce's philosophy of mathematics as having *some* affinities with logicism and *other* affinities with Intuitionism.

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