3-2007

On Logic in the Law: Something, but not All

Susan Haack

Ratio Juris. Vol. 20 No. 1 March 2007 (1–31)

On Logic in the Law: "Something, but not All"

SUSAN HAACK

Abstract. In 1880, when Oliver Wendell Holmes (later to be a Justice of the U.S. Supreme Court) criticized the "logical theology" of law articulated by Christopher Columbus Langdell (the first Dean of Harvard Law School), neither Holmes nor Langdell was aware of the revolution in logic that had begun, the year before, with Frege's *Begriffsschrift*. But there is an important element of truth in Holmes's insistence that a legal system cannot be adequately understood as a system of "axioms and corollaries"; and this element of truth is not obviated by the more powerful logical techniques that are now available.

1. An Old Story, and Its Relevance Today

In one of the best-known passages of *The Common Law*, Oliver Wendell Holmes writes:

> It is something to show that the consistency of a system requires a particular result; but it is not all. The life of the law has not been logic; it has been experience. The felt necessities of the time, the prevalent moral and political theories, even the prejudices which judges share with their fellow-men, have had a good deal more to do than the syllogism in determining the rules by which men should be governed. The law embodies the story of a nation's development through many centuries, and it cannot be dealt with as if it contained only the axioms and corollaries of a book of mathematics.[1]

What probably is not so well known is that the best-known sentence of this well-known passage—"The life of the law has not been logic; it has been experience"—had already appeared in print the year before, in Holmes's review of the second edition of Christopher Columbus Langdell's *Selection of Cases on the Law of Contracts*.

This book of Langdell's, Holmes writes, is "an extraordinary production,—equally extraordinary in its merits and its limitations." Chief

[1] Holmes (1881), in Novick, ed., 1995, 115.

among its limitations, according to Holmes, is that it is so focused on "the *elegantia juris*, or *logical* integrity of the system as a system" that it loses sight of the fact that "the law finds its philosophy not in self-consistency [. . .] but in history and the nature of human needs." Holmes manages to express both what he finds admirable in Langdell's work, and what he finds perverse, in one splendidly double-edged "compliment": Langdell, he writes, is "perhaps the greatest living legal theologian."[2] The following year, in a letter to Frederick Pollock, Holmes is even blunter: "A more misspent piece of marvelous ingenuity I never read [. . .] [Langdell] is all for logic and hates any reference to anything outside it"; he represents the "powers of darkness" in legal thinking.[3]

When Langdell and Holmes write of logic, the word they reach for is "syllogism." In 1880, when the second edition of Langdell's book was published, or in 1881, when Holmes's *The Common Law* appeared, neither would have been aware of the revolution in logic which had just begun—of the intellectual advances made by Gottlob Frege and Charles Sanders Peirce that were to supersede Aristotelian syllogistic logic and bring us, as we now say, "modern logic." For it would be many years before this work became known to a wider logical audience, and even longer before legal scholars began to take serious note of the new, modern logical techniques, and to try to apply them to questions about the law.

All this leaves one wondering: Are Holmes's criticisms of Langdell's "logical theology" sound? And, if they are, would they still apply, *mutatis mutandis*, to a post-Langdellian program revised in light of the far more powerful logical techniques now available? So after looking at Langdell's conception of the role of logic in the law and at Holmes's critique, I shall turn to some modern logical developments that might be thought relevant to the possibility of an updated Langdellian program, or something recognizably like it.[4]

The first strand of my argument will be that, while there are significant difficulties in Langdell's project that Holmes fails to notice, and significant weaknesses in some of the criticisms Holmes does make, there is nonetheless an important element of truth in Holmes's insistence that a conception of legal systems on the model of systems of "axioms and corollaries" is at best partial, and at worst misleading. And the second strand of my argument will be that, while the newer logical techniques may resolve some of the difficulties facing Langdell's project, they don't obviate this element of truth in Holmes's reservations, which is no less relevant to proposed applications of modern logic to legal theorizing than to Langdell's appeals to the syllogism.

[2] Holmes 1880, 233, 234.
[3] Holmes (10 April 1881), in Howe, ed., 1941, vol.1, 17.
[4] I was prompted to explore these issues when, working on an article on legal pragmatism (Haack 2005a), I came across Brewer 2000 and Grey 2000 (both in Burton, ed., 2000).

This is not to say that logic doesn't bear on the law at all; in due course I will touch on some of the many ways it does. But it is to say, echoing Holmes, that logic cannot reach the heart of the matter, that though "it is something [. . .], it is not all."

2. Langdell's Logical Theology of Law, and Holmes's Heresy

Langdell conceives of law as a "science," in the old sense in which the word means, roughly, "systematic body of knowledge." In the preface to the first edition of his *Selection of Cases on the Law of Contracts*—observing that, given the rapidly increasing number of reported cases, a comprehensive survey of all cases on contracts is hardly feasible—he writes that:

> Law, considered as a science, consists of a certain number of principles or doctrines. To have such a mastery of them as to be able to apply them with constant facility and certainty to the ever-tangled skein of human affairs, is what constitutes a true lawyer. . . . Moreover, the number of fundamental legal doctrines is much less than is commonly supposed. . . . If these doctrines could be so classified and arranged that each should be found in its proper place, and nowhere else, they would cease to be formidable in their number.[5]

According to this conception, at the core of a legal system is an interconnected congeries of key concepts which figure in a manageable number of key "doctrines" or "principles": e.g., truths about the nature of a legal promise, the central concept of the field of contracts. So mastery of the vast and ever-growing corpus of cases in any field of law requires a perspicuous analysis of its core concepts, by means of which apparently incompatible past decisions may be reconciled and the correct decisions in new cases deduced.

For example, applying his method of analysis and deduction in the opening pages of his *Summary of the Law of Contracts*, Langdell tells us that "[b]efore an act by the promisee, the so-called promise is in law only an offer [. . .]. It is not until it is accepted by the promisee that it becomes in law a promise." And, a couple of pages later: "As the performance of the consideration is what converts an offer into a binding promise, it follows that the promise is made, [. . .] at the moment when the performance of the consideration is completed. It also follows that up to that moment the offer may either be revoked, or destroyed by the death of the offerer, and the offeree thus deprived of any compensation for what he has done."[6]

On the basis of this analysis Langdell rejects the "mailbox rule," according to which a contract is made, before it has been received by the offeror,

[5] Langdell 1871, viii.
[6] Langdell 1880 (the "supplement" to Langdell 1879, published separately as a freestanding book), 1, 3.

as soon as the promisee puts his acceptance in the mailbox—a rule which, at the time he wrote, was settled law in England and in New York, but not in Massachusetts. The argument that supposedly justifies the mailbox rule is that if the contract must become known to the offeror the moment it is made, it must equally become known to the offeree the moment it is made; but a contract *inter absentes* can't become known to both parties at the same moment; and hence it need not become known to the offeror the moment it is made. But once the argument is thus "stated in the form of a syllogism," Langdell avers, it is seen to be unsound: "The fault is in the major premise, which is untrue."[7] The same analysis commits him to taking a hard line on the "flagpole problem": If X offers a reward of $100 to anyone who will climb to the top of this tall flagpole, but then—just as Y has laboriously climbed *almost* to the top—announces, "I revoke," then Y is not entitled to any compensation for his effort.[8]

Holmes objects that Langdell is "so entirely interested in the formal connections of things, or logic," that he ignores "the forces outside [the law] which have made [it] what it is, and without a grasp of which "[n]o one can ever have a truly philosophic mastery over the law."[9] He fails to see, Holmes continues, that judges' decisions are affected by the moral and political thinking of the day, by considerations of policy, and even by prejudice, far more than by logic:

> The life of the law has not been logic; it has been experience. The seed of every new growth within its sphere has been a felt necessity. The form of continuity has been kept up by reasonings purporting to reduce everything to a logical sequence; but that form is nothing but the evening dress which the newcomer puts on to make itself presentable according to the conventional requirements. The important phenomenon is the man underneath it, not the coat; the justice and reasonableness of the decision, not its consistency with previously held views [. . .]. The law finds its philosophy not in self-consistency, which must always fail so long as it continues to grow, but in history and the nature of human needs. As a branch of anthropology, law is an object of science; the theory of legislation is a scientific study; but the effort to reduce the concrete details of an existing system to the merely logical consequence of simple postulates is always in danger of becoming unscientific, and of leading to a misapprehension of the nature of the problem and the data.[10]

Holmes's best-known paper in legal philosophy, "The Path of the Law," appeared in 1896. Here we find him still arguing against "the notion that

[7] Langdell 1880, 19. The modern rationale for the mailbox rule would surely be very different: in part, that if a contract is to be made between parties not in one place at the same time, then one or the other must take the risk this imposes; and that—while we *could* assign the risk by flipping a coin—it is appropriate that it be borne by the offeror, who allowed the possibility of agreement by mail.
[8] Grey 1983, 15 (citing Langdell's discussion in Langdell 1880, 4).
[9] Holmes 1880, 234.
[10] Holmes 1880, 234.

the only force at work in the development of the law is logic"—which he now describes, apparently without irony, as a "fallacy."[11] He concedes that "in the broadest sense it is true that law is a logical development, like everything else"; meaning, I take it, that any reasonable account of law, like any reasonable account of anything, must be in some vague sense "logical." And he acknowledges that lawyers speak and write in the language of "analogy, discrimination, and deduction." But he continues to insist that, though "the language of judicial decisions is mainly the language of logic," and though judicial dissent is often faulted, "as if it meant simply that one side or the other were not doing their sums right," "[b]ehind the logical form lies a judgment as to the relative worth and importance of competing legislative grounds"; and it is this judgment, not the logical form, that is "the very root and nerve of the whole proceeding."[12] Moreover, Holmes continues, it is not only inevitable but desirable that over time judges' decisions gradually and incrementally adapt and modify old rules and precedents, the better to accommodate changed social circumstances and needs. "It is revolting to have no better reason for a rule of law than that so it was laid down in the reign of Henry IV," he writes, and "[i]t is still more revolting if the grounds on which it was laid down have vanished long since."[13]

It seems likely that by this time Holmes's target was not just Langdell (whom he doesn't mention explicitly in "The Path of the Law"), but also the judicial formalists who, though not exactly legal theologians of the pure Langdellian stripe, insisted on narrowly literal readings of the law.[14] Later, as a justice of the U.S. Supreme Court, Holmes would speak out many times against this kind of (quasi-logical) formalism:

> [G]eneral propositions do not decide concrete cases. The decision will depend on a judgment or intuition more subtle than any articulate major premise (1905).[15]

> [. . .] we must be cautious about pressing the broad words of the 14th Amendment [to the U.S. Constitution] to a drily logical extreme. Many laws which it would be vain to ask the court to overthrow could be shown, easily enough, to transgress a scholastic interpretation of one or another of the great guaranties in the Bill of Rights (1911).[16]

> [. . .] the provisions of the Constitution are not mathematical formulas having their essence in their form; they are organic living institutions [. . .]. Their significance is vital not formal; it is to be gathered not simply by taking the words and a dictionary, but by considering their origin and their line of growth (1914).[17]

[11] Holmes (1896), in Novick, ed., 1995, 396, 398.
[12] Holmes (1896), in Novick, ed., 1995, 397.
[13] Holmes (1896), in Novick, ed., 1995, 399.
[14] See White 1949, 59–75; Horwitz 1977, 253–66; and Horwitz 1992, 109–44.
[15] *Lochner v. New York*, 198 U.S. 545, 547 (1905), Holmes dissenting. Cf. Kelsen 1965; Kelsen 1967; Kelsen and Klug 1981; Ross 1968, 170.
[16] *Noble State Bank v. Haskell*, 219 U.S. 104 (1911), 110.
[17] *Gompers v. United States*, 233 U.S. 604, 610 (1914).

When we are dealing with words that are also a constituent act, like the Constitution of the United States, we must realize that they have called into life a being the development of which could not have been foreseen completely by the most gifted of its begetters .... The case before us must be considered in the light of our whole experience and not merely in that of what was said a hundred years ago (1920).[18]

Delusive exactness is a source of fallacy throughout the law (1921).[19]

And in other rulings we find Holmes making the thought explicit that legislation is not the only way for the law to adapt, or always the best way, that adjustments and readjustments through judicial reinterpretation are both inevitable and desirable. For example:

It does not seem to need argument to show that however we may disguise it by veiling words we do not and cannot carry out the distinction between legislative and executive action with mathematical precision and divide the branches into watertight compartments, were it ever so desirable to do so, which I am far from believing that it is, or that the Constitution requires (1928).[20]

In 1897, the year after Holmes's "The Path of the Law" appeared, Langdell (who became the first Dean of Harvard Law School in 1870, and Dane Professor of Law in 1880) gave an after-dinner speech entitled "Teaching Law as a Science" in which he reflected on legal education at Harvard. Here we find him still urging the importance of legal scholars' "follow[ing] the subtlest lines of reasoning, and detect[ing] the slightest flaws or sophistries in argument."[21] But we also learn more of his larger agenda, to "place the [American] law school [. . .] in the position occupied by the law faculties in the universities of continental Europe." But, he argues, if law were not, as he maintains, a science, there could be no justification for making it an academic subject of university teaching; if it were merely a "species of handicraft" it would be a skill best acquired, not from lectures and books, but by apprenticeship, and "the university [would] consult its own dignity in declining to teach it."[22]

Langdell's project suffers certain rather obvious weaknesses that Holmes doesn't mention. For one thing, Holmes doesn't point out how unsatisfactory the procedure by which Langdell arrives at his analysis of legal concepts seems to be. For example, Langdell tells us that what he offers is not an analysis of the concept of promise in general, but an analysis of the

[18] *Missouri v. Holland*, 252 U.S. 4126, 433 (1920).
[19] *Truax v. Corrigan*, 257 U.S. 312, 344 (1921), Holmes dissenting.
[20] *Springer v. Philippine Islands*, 277 U.S. 189, 211 (1928).
[21] Langdell 1887, 125.
[22] Langdell 1887, 123.

legal concept of a promise. At first blush it seems that he must mean "the concept to be discerned as common to all cases on contract"—something to be determined, presumably, by surveying the relevant cases and looking for the conceptual core common to them all. But then, remembering that in 1880 the mailbox rule was settled law in some states but not in others, you wonder: "'The' legal concept?—*in what jurisdiction*: in New York, in Massachusetts? And 'the' legal concept *when*?—in 1880, in 2006?" And when Langdell avers that certain cases were wrongly decided *because the court got the legal concept of a promise wrong*, he implicitly acknowledges that there *is no* single, coherent concept common to all cases. If so, however, what standard can he be employing when he determines which cases were correctly decided, and which incorrectly?[23]

Nor does Holmes ever ask how, exactly, the key "principles" and arguments in Langdell's science of contract law are to be represented in the language of Aristotelian logic, or notice that Langdell's statement of the argument for the mailbox rule "in the form of a syllogism" is not, in fact, in anything like that form. These are likely to strike a logician as significant omissions, not least because the concept of a promise is essentially relational: A promises B, in return for consideration C, to do X; but, as logicians would have been aware by the time Holmes and Langdell wrote, Aristotle's logic is unable to handle arguments essentially involving relations.

It might be thought, also, that the criticisms Holmes does level against Langdell's conception of law as science rest on a very uncharitable reading. For in fact Langdell had not denied, but had expressly acknowledged, that "each doctrine or principle," i.e., each of the truths about the concepts at the core of a legal system, "has arrived at its present state by slow degrees; [. . .] it is a growth, extending in many cases through centuries."[24] Furthermore, he had presented his logical method not as a descriptive account of how judges do in fact decide cases, but as prescribing how cases *ought* to be decided. And he had not assumed that there must be a single, coherent concept discernable in common to all decided cases in a field, but had noted inconsistencies among earlier cases—and, as we saw, sometimes dismissed this or that past case as simply mistaken (the footnote accompanying his argument that a contract may be revoked up to the moment consideration is completed, for example, reads: "Offord v. Davis, 12 C.B.

[23] In Dickinson 1929, John Dickinson suggests that the "inductive" theory of law adopted by Sir Frederick Pollock in England and W. G. Hammond in the U. S. "seems to be have been one of the presuppositions underlying the teaching methods of Langdell" (141), but stops short of describing Langdell's legal *theory* as inductive (Langdell introduced the case-study method to Harvard Law School). In Grey 1983, Thomas Grey notes that Langdell's theory seems at first blush to be "an incomprehensible jumble of induction with deduction and of norm with fact," and suggests that we might try to make sense of it on the model of J. S. Mill's conception of geometry (16). I fear it really *is* a jumble.

[24] Langdell 1871, viii.

n.s. 748, Cas. on Contr. 33. Bradbury v. Morgan, 1 H.&C. 249, cited in Offord v. Davis, *supra* is *contra, but it must be deemed erroneous*").[25]

But for all its faults both of commission and of omission, there is something in the tone and tenor of Holmes's critique that seems both true and important. First: Holmes sees that a serious acknowledgment of the contingent, historically conditioned character of legal systems, of how they evolve and adapt to changed social circumstances over time, of how judges interpret and reinterpret statutes, rules, and precedents in the light of current moral and political ideas and the myriad Pushmepullyou social pressures to which any legal system is subject, would reveal the implausibility of the idea that legal concepts have some fixed, stable, "real" meaning to be discovered—intuitively, inductively, or any other way.

Second: Holmes acknowledges the desirability of consistency; meaning, I take it, not primarily consistency in the precise but narrow formal sense that concerns a logician, but consistency in the subtler, broader sense that concerns lawyers and legal theorists.[26] Of course, even with formal inconsistencies, logic alone won't tell you what adjustments should be made to a system; and with the kind of inconsistencies that concern a jurist logic has even less to say. Treating like cases alike is an important element of justice; but logic won't tell you what cases are alike in legally relevant ways. Predictability in a legal system is of value to society; but logic won't tell you when predictability should be overridden by other values. Holmes sees that it is not just a brute historical fact that legal systems gradually shift and change, or that legal concepts are contested, expanded, and refined in response to competing social interests and evolving social needs; it is a good thing—for helping to resolve those inevitable tensions, and to accommodate those evolving needs, is what a legal system is *for*.

Not surprisingly, Holmes doesn't say, as I might, that logic represents the joints of discourse, not its heart; or that, being topic-neutral, syncategorematic,[27] it isn't *all* of *any* matter. But he does seem to have grasped, even if he doesn't fully articulate, the enormously important insight that rationality calls not for rigidity, but for flexibility, adaptability—in the law as elsewhere.

When Langdell insists that law *must* be a science, or else there would be no legitimate place for a law school in a university, it's almost as if he is responding in anticipation to Thorstein Veblen's dry comment that "in point of substantial merit the law school belongs in the modern university

[25] Langdell 1880, 3, n.3, final italics mine.

[26] A set of sentences of a formal language is formally, or logically, inconsistent if and only if a contradiction—the conjunction of a formula and its negation—can be derived from it. On broader and narrower conceptions of consistency and related concepts, see Haack 2004.

[27] "*Syncategoremata*" are "words such as 'and,' 'or,' 'not,' 'if,' 'every,' 'some,' 'only,' and 'except' which cannot function as terms but are of special importance in logic because they show the forms of statements": Kneale and Kneale 1962, 233.

no more than a school of fencing or dancing."[28] When Holmes insists that system, consistency, *elegantia juris*, are not the only or necessarily the overriding values in legal decision-making, it's almost as if he is chiming in with Ralph Waldo Emerson's famous observation that "a foolish consistency is the hobgoblin of little minds, adored by little statesmen, philosophers, and divines":[29] "Yes; *and* by those annoying legal theologians!" But neither Langdell nor Holmes sounds much like a logician.

Neither seems to have had more than a layman's grasp of what logic is and does. Langdell presents his arguments entirely informally—indeed, the reader begins to suspect that when he writes of "syllogism" it is little more than a vague synonym for "deduction"; and Holmes's critique includes no logical specifics. Most to the present purpose, neither shows any awareness that by the turn of the nineteenth century the old syllogistic logic with which they apparently had at least a passing acquaintance was being superseded by a newer and far more powerful logical apparatus.

Legal theorists of a Langdellian cast of mind now have a whole battery of powerful new formal tools at their command. Nevertheless, I shall argue, the old debate between Langdell and Holmes remains of more than merely antiquarian interest; for while these new logical tools certainly have something to contribute, they by no means nullify the insight behind Holmes's resistance to the idea of law-as-logical-system. Before I can develop this argument, though, the first task is to look at some of these modern logical innovations, and some of their proposed applications in legal contexts.

3. The Revolution in Logic, and Its Aftermath

An initial difficulty is that the word "logic" is used in more than one way. In a very common ordinary-language usage, "logical" is in effect an honorific adjective roughly equivalent to "reasonable, sensible." "There has to be a logical explanation," we say, meaning "there has to be a reasonable explanation"; and "that's logical," meaning "that makes sense." (This is how Holmes is using the word when he concedes that any reasonable account of legal phenomena, as of any phenomena, must be "logical.") But in an old philosophical tradition going back at least to Aristotle, logic has been conceived as a theoretical discipline: the theory of whatever is good in the way of reasoning. Logic, so understood, is a normative enterprise—as distinct, most importantly, from a descriptive study of how people actually reason. "Good," in such a theory, is focused on the avoidance of contradiction and on validity, the capacity of an argument to preserve truth—as distinct, most importantly, from its persuasiveness to

[28] Veblen (1918), reprinted 1954, 211.
[29] Emerson (1841), in Atkinson, ed., 1940, 152.

this or that audience. In this usage, "logic" encompasses both systematic, formal representations of valid arguments and theorems, and informal, philosophical explorations of such concepts as *term*, *proposition*, *argument*, *truth*, etc..

Recently, however, the word has come to be more often used in a narrower sense in which it refers to those formal logical systems exclusively; philosophical explorations of key logical concepts are now more likely to be classified, rather, under the rubric "philosophy of logic." The "revolution in logic" that concerns us here was a revolution in formal deductive logic.[30] "Formal" means "syntactic, expressible in terms exclusively of the shape or structure of sentences in a formal language ('well-formed formulae'), regardless of their content." "Deductive" means "intended to capture the class of necessarily truth-preserving inferences, and of formulae true in all interpretations of their non-logical constants."

Briefly and roughly: Aristotle's formal logic, the logic of such familiar syllogistic schemata as "All As are Bs; All Bs are Cs; therefore, All As are Cs," was a kind of class calculus. Subsequently, Stoic logicians had explored propositional logic, in which such arguments as "If p then q; p; therefore, q" can be represented.[31] A key step on the way to modern logic was George Boole's logical algebra, which could be interpreted as a calculus of probabilities, as a calculus of propositions, or as a calculus of classes.[32] By 1880—independently, by different routes, and in different notations—Frege[33] and Peirce[34] had developed a new formal logic, unlike anything that went before: a logic in which propositional and predicate calculus are unified, and which can represent the validity of arguments essentially involving relational predicates.[35]

Unlike Aristotelian syllogistic, which can represent only the monadic properties expressed by one-place predicates like "horse" or "animal," the Frege-Peirce logic can also accommodate relations, i.e., polyadic properties expressed by many-place predicates like "__ is taller than __ ." So it can represent the validity of such arguments as "All horses are animals; therefore, everything that is the head of a horse is the head of an animal"; and, much more importantly, of mathematical arguments involving such relational predicates as "larger than," "successor of," etc. (and—yes!—legal

[30] Rather as "xerox" has become a generic word for photocopying, and "kleenex" for tissues, this shift in the use of "logic" was probably in part a response to the success of this revolution. I'm not sure the shift was desirable; nevertheless, for convenience I shall henceforth be using the word "logic" in the narrower modern sense, unless otherwise indicated.

[31] See Bocheński 1961, 105–133; Kneale and Kneale 1962, 113–76.

[32] Boole (1854), reprinted 1958.

[33] Frege 1879, English trans. in Bynum, ed., 1972. On Frege's advances over Boole, see Dudman 1976.

[34] In Hartshorne *et al.*, eds., 1931–58, 31.154–251 (1880).

[35] On the influence of Frege's and Peirce's work on subsequent developments, see Putnam 1982.

arguments involving such relational predicates as "__ promised __, in return for consideration __, to __"). And unlike Boolean algebra, which may be interpreted either as a propositional or as a class calculus, but not as both simultaneously, the Frege-Peirce logic *unifies* propositional and predicate calculus.

Propositional calculus represents valid arguments and logical truths expressible in terms of simple ("atomic") propositions, and compound ("molecular") propositions formed from these by means of extensional, truth-functional operators like "and" and "not"—e.g., the argument from "p and q" to "p"—but doesn't concern itself with the internal structure of propositions. Then predicate calculus, or "quantification theory," builds on this to include representations of valid arguments and logical truths that do depend on the internal structure of propositions. This internal structure is analyzed in terms of quantifiers (such as "for all x," "for some x"), singular terms or names ("a," "b," etc..) and predicates, as in "whatever is F is G," i.e., "for all x, if Fx then Gx"; so it becomes possible to represent arguments like "For all x, either Fx or Gx" and "Not Fa" to "Ga." This Frege-Peirce logic, canonically expressed in 1910 in Russell and Whitehead's *Principia Mathematica*, was a major intellectual advance. By now, it has come to be known as "classical logic" (and Aristotelian logic, to which that label used to be applied, is more likely to be called "traditional"). As we will see, it has been used by legal scholars primarily as a tool for resolving structural ambiguities.

Not surprisingly, the success of the new logic spurred many further developments.

Some logicians proposed systems which restrict the set of logical truths and/or valid inferences recognized by classical logic—deviant logics.[36] Many-valued logics, for example (often associated with the work of Łukasiewicz and Post in the early 1920s,[37] but developed well before that by Peirce, who had experimented with "triadic logic" as early as 1909)[38] allow intermediate truth-values or truth-value gaps, and generally repudiate, *inter alia*, the law of excluded middle ("p or not-p"). Intuitionist logics (developed by Dutch mathematician L. E. J. Brouwer and his followers)[39] also suspend the law of excluded middle but, unlike many-valued systems, are not truth-functional. At the extreme of deviance, the "fuzzy logic" devised by electrical engineer Lotfi Zadeh, constructed on an infinitely-valued base logic, is supposed to be, not just a logic of vagueness,

[36] On deviant logics generally, see Haack 1974; Haack 1996; and Haack 1978, chapters 9 and 11.
[37] Łukasiewicz (1920) and Łukasiewicz (1930) in McCall, ed., 1967. Post (1921) in van Heijenhoort, ed., 1967. See generally Rescher 1969.
[38] Peirce, *Logic Notebook* (February 23rd, 1909), in Haack, ed., 2006, 217–20 (transcribed by assoc. ed. Robert Lane).
[39] Brouwer 1952; Heyting 1956; Heyting 1966.

but a logic which is itself vague, requiring fuzzy truth-values and vague, informal rules of inference.[40] Curiously, the possibility of using some kind of "gappy" logic to represent legal gaps doesn't seem to have been explored; but fuzzy logic (or "logic," as I might be inclined to say) has attracted the attention of some evidence scholars.

Other logicians proposed systems which expand the classical set of valid inferences and logical truths by adding to classical logic new, non-extensional ("intensional") vocabulary, and new axioms or theorems involving that vocabulary—extended logics.[41] Modern modal logics (usually traced to C. I. Lewis's work in the early twentieth century and the work of those who subsequently built on and extended it,[42] but developed well before that by Peirce, in the "gamma graphs" of 1903)[43] add the non-truth-functional operators "necessarily" and "possibly," and various stronger and weaker axioms governing these operators. Epistemic logics introduce, by analogy with modal operators, "x knows that" and "x believes that." Tense logics introduce "It was the case that" and "It will be the case that."[44] Deontic logics introduce "obligatory," "permitted," and "forbidden" (the analogue of "impossible" in modal logic).[45] At the outer limits of extended logic, John Woods proposes a modal operator, "Once upon a time," to play a role in his "logic of fiction."[46] Not surprisingly, it is deontic logic that has attracted most attention from legal theorists hoping for a rigorous formal representation of the structure of legal orders.

Relevance logicians challenged the classical conception of validity as necessary truth-preservation: which, they argued, is far too weak, because it acknowledges as valid arguments in which the premisses are in no way relevant to the conclusion.[47] For example, in classical logic anything and everything follows from a contradiction: The argument from "p and not-p" to "q" is valid no matter what q may be (since it is impossible for its premiss to be true, it is *a fortiori* impossible for its premiss to be true and its conclusion false). Anderson and Belnap's system R of relevance logic is both extended and deviant, introducing new vocabulary in the form of a

[40] Zadeh 1975. As I observed in Haack (1981), reprinted in Haack 1996, 237, "it would scarcely be an exaggeration to say that fuzzy logic lacks every feature that the pioneers of modern logic wanted logic *for*."

[41] On extended logics generally, see Haack 1978, chapter 10.

[42] Lewis 1912; Lewis 1918; Lewis and Langford (1932), second edition 1959, 153–66. See generally Hughes and Cresswell 1968.

[43] In Hartshorne *et al.*, eds. 1931–58, 4.409–13 (1903), 463–72 (c.1903), 511 ff. (1903), 576 ff. (1906).

[44] Prior 1957; Prior 1967; Prior 1968. See also Haack 1978, 156–62.

[45] Von Wright 1950; Von Wright 1951; Von Wright 1963.

[46] Woods 1974. Woods calls this modal operator "the olim operator," "*olim*" being Latin for "once upon a time."

[47] On relevance logic, see Anderson and Belnap 1975; Anderson, Belnap, and Dunn 1992. On the somewhat different enterprise known as "relevant" logic, see Routley *et al.* 1982; Norman and Sylvan (formerly Routley), eds., 1989.

connective for "relevant implication" and repudiating *modus ponens*, the classically valid principle of inference from "p → q" and "p" to "q" (where "→" is the classical truth-functional operator "material implication"). So far as I am aware, however, legal scholars have not been attracted to the idea of applying relevance logics in, for example, analysis of evidence, a field of legal studies where relevance is a key notion—not without reason, probably, given that these logicians' treatment of relevance is, of necessity, entirely formal.

Other logicians have explored the larger family of "paraconsistent" systems, which allow inconsistencies (i.e., include theorems such that one is the negation of anther), and yet are not trivial (not all wffs of the system are theorems of the system).[48] Graham Priest presents his "dialethic logic," in which some contradictions are true, as radically rivalling classical logic.[49] But the founder of paraconsistent logic, Newton da Costa, writes that while these system may be seen either as rivals or as complements to classical logic, depending on the philosophical position one takes, his stance is neutral: "Paraconsistent logic, for me, is a discipline mathematical in spirit."[50] The various proposed extensions of such systems include *inter alia* paraconsistent deontic logics, including proposals regarding "legal modalities."[51]

There have also been efforts to apply the methods of formal logic beyond the class of truth-capable declarative sentences, to sentences in the imperative mood, for example, or to questions—non-declarative logics. Since non-declarative sentences are not truth-valued, some analogue of validity couched in other terms than truth-preservation will be needed: For "imperative logic," for example, satisfaction has been proposed as the analogue of truth, and satisfaction-preservation as the analogue of validity. However, imperative logic turned out to be something of a damp squib; and legal theorists have mostly preferred to explore the possibilities of deontic logic.[52]

The successful formalization of deductively valid arguments has understandably inspired the thought that weaker but still worthy arguments (arguments which, while not necessarily truth-preserving, nevertheless give some degree of support to their conclusions) might be susceptible of formal treatment—non-deductive logics. Once again, Peirce was a pioneer, developing a threefold division of kinds of reasoning: While in deductively

[48] da Costa 1974; Arruda 1980; Priest *et al.* 1989; da Costa 1997.
[49] Priest 1987.
[50] Newton da Costa, e-mail message to SH (12.25.05).
[51] Puga and da Costa 1987; Puga, da Costa, and Vernengo 1992. Bazhanov 1990 suggests that anticipations of paraconsistent logic can be found in N. A. Vasiliev's work as early as 1910.
[52] There is a hint, but not much more than that, of a role for imperative logic in a very early paper in the "logic of law" *genre*, Oppenheim 1944. In Ross 1968, while noting that imperatives are the most obvious linguistic form for expressing directives (36), in due course Alf Ross concentrates his attention on deontic logic (139–81).

valid arguments the truth of the premises guarantees the truth of the conclusion, in inductive arguments (involving inferences from observed instances to generalizations) the connection between premises and conclusion is weaker, and in abductive arguments (involving inferences from observed phenomena to possible explanations) it is weaker yet. Peirce extended this threefold categorization to include arguments by analogy, construed as combining elements of all three.[53] Since Peirce's time, numerous styles and systems of inductive logic have been developed. Abduction, by contrast, has often been relegated to the "context of discovery," generally taken to be the sphere of psychology or sociology rather than of logic. Some legal scholars, however, have suggested that this might be the mode of reasoning involved in working out the general legal principle behind a range of cases.[54] Analogy has been a topic of enduring interest to students of legal reasoning; it has often been treated as *sui generis*, but sometimes, as we shall see, in a quasi-Peircean way.

Frege's logical innovations were motivated primarily by the desire to work out a system adequate to represent the conceptual structures and arguments of mathematics. Peirce—the first person to describe himself in *Who's Who* as a logician—was primarily focused on the methods of the sciences. More recently, proponents of the many and various deviant logics have proposed applications to metaphysical issues about future contingent events or non-existent "objects," to vague statements, to the understanding of quantum mechanics, and so forth; and, with fuzzy logic, to the resolution of problems in electrical engineering. But of course I shall focus here on some of the proposed application of modern logic, classical and non-classical, deductive and non-deductive, to issues in legal theory.

4. The New Logical Theology, and Haack's Heresy

As Bentham's marvelous metaphor of "Injustice, and her handmaid Falsehood" reminds us,[55] factual truth is an essential element of substantive justice; it really matters that the person who is punished be the person who actually committed the crime or caused the injury. And there are interesting and substantive issues about the role of logic in determinations of factual truth. They are, however, quite general epistemological issues, with no peculiarly intimate relation to philosophy of law. Nevertheless, it is

[53] For example, in Hartshorne *et al.*, eds., 1931–58, 1.65–74 (c.1896), 2.619–44 (1878), 5.151–79 (1903); and, on analogy specifically, 2.733–4 (1883). Note, however, that though he distinguished three types of reasoning as early as 1869, Peirce later refined the distinction; and around 1910 wrote that "in almost everything I printed before the beginning of the century I more or less mixed up Hypothesis and Induction" (8.227, c. 1910).

[54] See, e.g., Tuzet 2005.

[55] Bentham (1827), reprinted 1978, vol. I, 22.

worth speaking to them briefly here, not least because this will suggest some morals applicable in legal theory too.

Logical Positivism was so-called (by contrast with older forms of plain Positivism) because of its reliance on the new formal logic; and—dismissing most of traditional philosophy as cognitively meaningless, or at best bad poetry—took the remaining legitimate philosophical task to be the articulation of the "logic of science." Not surprisingly, then, for much of the twentieth century philosophers of science—inductivists like Reichenbach, Carnap, and Hempel as well as deductivists like Popper—took for granted that if science is a rational enterprise, its rationality must be explicable in narrowly formal-logical terms.[56] And so did the more recent small army of radical sociologists of science who, sensing that these positivist and post-positivist attempts to develop a plausible "rational reconstruction" of the logic of science weren't working, came to deny or denigrate the supposed rationality of science.

However, as I have argued in detail elsewhere, this shared assumption is mistaken. Fallible, fumbling, and imperfect as it is, scientific inquiry is a rational enterprise; but the epistemological burden that formal logic, however sophisticated, can be expected to carry in explaining what that rationality amounts to is really quite modest. An adequate account of what makes the evidence with respect to an empirical claim stronger or weaker, what makes such a claim more or less warranted, or what makes empirical inquiry better or worse conducted, cannot be couched exclusively in narrowly logical terms, but must be "worldly"; i.e., it must encompass inquirers' interactions with particular things and events in the world, and the relation of language to kinds of thing and phenomena. (So shifts and changes in the language of science are not, as many philosophers of science of the twentieth century supposed, inevitably an impediment to rationality, but often contribute to intellectual advance.) If this is right, while formal-logical apparatus will doubtless have a part to play, a complete account of the supportiveness of evidence, the warrant of claims etc., will also require additional resources of a very different, non-formal kind.[57]

In the legal literature on the "new evidence scholarship" (so-called because it focuses not, like traditional evidence scholarship, on legal rules of procedure and admissibility, but on the substantive analysis of evidence)[58] there have been many and various proposed applications of

[56] As others did subsequently, after the publication of Kuhn 1962: e.g., the structuralists who hope to sustain the rationality of science by construing scientific theories as complex set-theoretical predicates, and hence as neither true nor false, but as being, or not being, a model of the world. See Stegmüller 1976.

[57] See Haack 1993a, especially chapter 4; Haack 2003, especially chapter 2 (on the "Old Deferentialism" in post-positivist philosophy of science), chapter 3 (on the nature of evidence and the determinants of evidential quality), chapter 7 (on those radical sociologists of science), and chapter 8 (on the growth of scientific language); and Haack 2005b.

[58] See Lempert 1988.

logical machinery—or, more often, quasi-logical formal machinery: Wigmore diagrams,[59] "defeasibility logics,"[60] even fuzzy logic;[61] and, of course, there have been many and various proposed applications of the mathematical calculus of probabilities.[62] These, however, are less to the present purpose than proposed applications of the languages and methods of modern logic more specific to legal theory.

The range of such proposed applications is very broad, from quite modest suggestions of ways in which modern logical formalism might be helpful in resolving ambiguities in legal instruments, through ideas about how to formalize what are taken to be characteristically legal styles of reasoning, to ambitious-sounding proposals regarding the "logic of the law" quite reminiscent of Langdell. A full sweep is hardly feasible, so I shall look quite briefly at a few (I hope, characteristic) examples:[63] the first involving applications of classical deductive logic, the second an application of extended (deontic) deductive logic, and the third an application of non-deductive logic. These will confirm, as the history of narrowly logical models of rationality in science suggests, that formal logic can play only a quite modest role in a full understanding of what is reasonable or rational.

Likening natural languages to the human hand, and the language of his newly-invented formal logic to specialized tools like hammers or wrenches, Frege had especially stressed that, though the flexibility and versatility of natural languages is useful for many purposes, their susceptibility to ambiguity and equivocation makes them unsuitable for the rigorous representation of arguments. "We build for ourselves artificial hands, tools for particular purposes, which work with more accuracy than the hand can provide," he wrote; and what makes this increased accuracy possible is "the very stiffness and inflexibility of parts the lack of which makes the hand so dextrous."[64]

[59] Wigmore (1913), reprinted 1988, 751–60. There is a summary in Anderson *et al.* 2005, 123–44.

[60] See, e.g., Prakken 2004.

[61] Zadeh 2002.

[62] I was interested to learn that *Jurimetrics*, which has a markedly Bayesian cast, is the successor-journal of the now-defunct *Modern Uses of Logic in the Law*. Tribe 1971 remains the classic critique of the excesses of what one might call "legal probabilism." I have expressed my reservations about the epistemological usefulness of the mathematical theory of probabilities in Haack 2003a, 73–7; Haack 2003b, 213–4; and in Haack 2005b.

[63] I shall not, however, look at the arguments of those who, in a curious twist on Langdell's optimism that logical analysis would enable the legally correct decisions to be deduced, have claimed that the application of Gödel's incompleteness theorem confirms the idea of law as essentially indeterminate. This seam of recent literature is summarized in Dow 1993.

Hao Wang reports that, in his interview for American citizenship (in preparation for which he had carefully studied the U.S. Constitution), Gödel explained to Judge Philip Forman that he had discovered a loophole in the Constitution that allowed the possibility of transforming the U.S. into a dictatorship. See Wang 1987, 115–6.

[64] Frege (1882), English trans. in Bynum, ed., 1972, 86.

Some proponents of a relatively modest role for logic in the law have stressed this capacity to reveal and resolve syntactic ambiguities. In 1950, John Pfeiffer recounted in the pages of the *Scientific American* how the Prudential Life Insurance Company had used formal-logical tools to rewrite a provision in a contract, and explained that "lawyers have begun to call on mathematicians to go over their contracts," which:

> [. . .] may run into many pages of fine print packed with stipulations, contingencies and a maze of ifs, ands, and buts. Are the clauses worded as simply as they might be? Are there loop-holes or inconsistencies? A symbolic analysis can readily answer such questions [. . .].[65]

In 1955, stressing the superior expressive power of modern logic over the language of the syllogism, Ilmar Tammelo offered symbolic analyses of various legal concepts.[66] In 1957, under the bold title "Symbolic Logic: A Razor-Edged Tool for Drafting and Interpreting Legal Instruments" Layman Allen argued that "a new approach to drafting, using certain elementary notions of symbolic logic, can go a long way toward eliminating [. . .] inadvertent ambiguity" in statutes, contracts, wills, conveyances, and regulations.[67] In 1962 James C. Miller used (quasi-)logical techniques to disambiguate provisions in Article 6 (b) of the Nuremburg Tribunal and in the Iranian Oil Nationalization Act of 1951.[68] These were relatively unsophisticated early efforts; but the underlying idea seems perfectly unobjectionable.[69]

Formal tools can indeed be very helpful in cleaning up the kind of ambiguity that depends on structure: the scope of a quantifier,[70] for example, or the difference between inclusive and exclusive disjunction, or between implication and mutual implication—in the law as elsewhere. However, such syntactic ambiguities are not the, or even a, main source of legal indeterminacies, which derive mostly from the open-texture or vagueness (or sometimes the semantic ambiguity) of key concepts such as

[65] Pfeiffer 1950, 22, 23–4.
[66] Tammelo 1955.
[67] Allen 1957, 833. Unlike Tammelo (whose name, incidentally, he misspells "Tammello"), Allen uses no logical apparatus more complex than elementary, classical propositional calculus.
[68] Miller 1962.
[69] I will also mention (though I can't discuss it here), the enterprise linguists call "dynamic logic," which offers a formal treatment of anaphora and cross-reference among singular terms, and which might conceivably help clarify issues about cross-reference in legal instruments. Groernendijk and Stokhof 1991; Van Eijk and Kamp 1997.
[70] For example, as in: "There is a loophole in every contract," which might be read *either* (i) as "for all x, if x is a contract then there is a y such that y is in x and y is a loophole," which would be formally represented along the lines of: "(x) (Cx → (Ey) Ryx and Ly)"; *or* (ii) as "there is a loophole which is in every contract," which would be formally represented along the lines of: "(Ey) (Ly & (x) Cx → Ryx)," and implies that the same loophole is in every contract.

"responsible," "sane," "privacy," "causation," etc.. In short, applications of logical techniques to resolve syntactic ambiguities are clearly something, but not all.

Tammelo also ventures (in the latter part of the paper discussed earlier, and in subsequent books) into a considerably more ambitious project he calls "legal logistic": a project which calls on the resources of deontic logic to develop an account of the structure of normative systems in general, and of legal systems in particular.[71] But I will take Carlos Alchourrón and Eugenio Bulygin's *Normative Systems*[72] as my exemplar of this approach[73]—in part because it is a more sophisticated example of the *genre*, logically and philosophically, than Tammelo's *Modern Logic in the Service of Law*, and in part because Thomas Grey borrows from it in his exegesis of Langdell's project as providing "a somewhat more precise description of Langdell's classical orthodoxy."[74]

Of course, Alchourrón and Bulygin are well aware of the superior expressive power of modern logic over the syllogism.[75] Moreover, at least on the surface it is less of a mystery what the source of their "primitive sentences" (the analogue of Langdell's "doctrines" or "principles") is supposed to be. For they are at pains to distance themselves from a pure rationalism: The fundamental principles of a normative system of law, they tell us very plainly, are to be "selected" empirically, by reference to legislation, to legal codes, and to precedents. "Legal science," as they conceive it, like the natural sciences, is partly "rational" (they mean, partly formally-logical), and partly empirical. But the specifically philosophical task, as they conceive it, is to supply a "rational reconstruction" of the logical framework of normative systems—just as philosophers of science of that period aimed to do for physics, etc.[76]

Alchourrón and Bulygin's conception of the philosophical task, like the phrase "rational reconstruction," is emblematic of the time, 1971, and of the series, "The Library of Exact Philosophy," in which their book appeared. So it will be prudent to keep in mind the warning that one of the most important proponents of logical method in philosophy of science, Rudolf Carnap, once issued to himself: That such projects run the risk of producing "a theory which is wonderful to look at in its exactness, symmetry and formal elegance, yet woefully inadequate for the task [. . .] for which it is

[71] Tammelo 1969; Tammelo 1978.
[72] Alchourrón and Bulygin 1971.
[73] But I will also mention (though I can't discuss it here) Carlos Alarcón Cabrera's argument that there are inconsistencies in the Spanish constitution. Alarcón Cabrera 1993; Conte and Alarcón Cabrera 1995; Conte and Alarcón Cabrera 1999; da Silva 2004.
[74] Grey 1983, 6 ff..
[75] Alchourrón and Bulygin 1971, 82.
[76] Alchourrón and Bulygin 1971, 53 (on rational versus empirical sciences) and 7–9 (on "rational reconstruction").

intended."[77] At the head of the first chapter of their book, Alchourrón and Bulygin cite R. M. Martin: "The methods of formalization and interpretation are deeply engrained in the western mind and perhaps constitute the ideal prototype of some aspects of what we call *rational* thought";[78] two pages later, they paraphrase Martin thus: "The method of rational reconstruction reflects an essential aspect of all rational thought."[79] But the reader familiar with Holmes's critique of Langdell or with Carnap's warning is likely to hear in Martin's "*perhaps* the ideal prototype of *some* aspects" a hint that over-emphasizing the logical structure of a legal system runs the risk of neglecting the deeply socio-historical character of legal systems and the dynamics of legal shifts and changes. I fear that Alchourrón and Bulygin fall into this trap.[80]

They acknowledge, for example, that the identification of "primitive sentences" raises questions about "the operation which jurists refer to under the vague term 'interpretation.'" In view of this rather dismissive comment, perhaps it's not surprising that their (very brief) discussions are focused primarily on whether changes in interpretation should be classified as changes in the primitive sentences or as changes in the rules of inference of the system; nor that these discussions throw no light on the character of or reasons for legal interpretation, or on what might make one interpretation preferable to another.[81] This is why I said earlier, very guardedly, that it "seems, at least" to be less of a mystery how Alchourrón and Bulygin arrive at their primitive sentences than it is how Langdell arrives at his "doctrines": For without answers to these questions about interpretation it is quite unclear what the content of those primitive sentences should be.

Again: Alchourrón and Bulygin acknowledge that identifying *all* the valid sentences at the basis of any legal order would be an overwhelming task, and propose to avoid this difficulty by focusing, not on legal systems in their entirety, but on those parts of a legal system that include sentences relevant to a given class of legal issues. But they have nothing to say about how these classes of sentences are to be identified: which, however—given the vagaries of "relevant"—is to say the least a formidably difficult question.

And again: Alchourrón and Bulygin note that legal systems shift and change, that "the different sources of law are constantly producing new sentences that have acquired validity, while other sentences lose their

[77] Carnap 1962, 218.
[78] Martin 1958 (cited in Alchourrón and Bulygin 1971, 7).
[79] Alchourrón and Bulygin 1971, 9.
[80] Alchourrón and Bulygin focus on the Argentinian legal system. So if my criticisms are correct they confirm that, as I believe, while logic may have a slightly larger role to play in an understanding of civil-law than of common-law systems, it cannot constitute a full account of even the most codified legal regimes.
[81] Alchourrón and Bulygin 1971, 67, 91.

validity by derogation or desuetude"; and propose to accommodate this complication by focusing on time-slices of those parts of what would more ordinarily be called "the French legal system," "the Argentinian legal system," etc.—"*momentary* systems," as they say.[82] But this purely formal accommodation of legal change seems far from the heart of the matter.

Let me illustrate by reference to a real-life example from the history of Florida evidence law. In *Kaminski* (1952),[83] specifically with respect to polygraph evidence, the Florida Supreme Court endorsed the test of admissibility known as the *Frye* Rule: a test derived from a 1923 D.C. case, requiring that, to be admissible, the "principle or discovery" from which novel scientific testimony is derived must be "sufficiently established to have gained general acceptance in the field to which it belongs."[84] In *Coppolino* (1968),[85] the 2nd District Court of Appeals noted that Florida had apparently adopted *Frye* as the rule governing the admissibility of novel scientific testimony generally—but nevertheless held it within a court's discretion to admit novel scientific testimony even if it *didn't* satisfy *Frye*.

In 1975 the Federal Rules of Evidence were enacted. Rule 702 required that expert testimony be relevant (and not legally excluded under rule 403 on grounds of waste of time or misleading or confusing the jury); but it didn't mention general acceptance in the relevant community. In 1978 the Florida Evidence Code was adopted; it follows the Federal Rules of Evidence quite closely, and there is no reference to "general acceptance" in its Rule 702 either. In *Kruse* (1986)[86] the 4th District Court of Appeals described Florida evidence law on this subject as more flexible than the old *Frye* test, but as still requiring some indication of reliability of the subject-matter of scientific testimony.

In 1993 federal law on the subject of expert testimony was modified when the U.S. Supreme Court ruled in its first-ever scientific testimony case, *Daubert v. Merrell Dow Pharmaceuticals*,[87] that *Frye* had been superseded by the Federal Rules of Evidence; but added that courts had a responsibility to screen proffered scientific testimony not only for relevance but also for reliability. The same year, in *Flanagan*,[88] the Florida Supreme Court insisted that, in Florida, novel scientific testimony must still meet *Frye*; and in *Brim* (1997) that "[d]espite the federal adoption of a more lenient standard, we [FLA] have maintained the higher standard of reli-

[82] Alchourrón and Bulygin 1971, 77. Oppenheim had anticipated the need for this fragmentation into momentary systems. Comparing law with motion pictures, he writes: "We can stop the projector at any time and concentrate our attention on the picture reflected on the screen at that moment" (Oppenheim 1944, 143).
[83] *Kaminski v. State*, 63 So.3d 339 (1952).
[84] *Frye v. U.S.*, 54 App.D.C. 46, 293 F. 1013, 1014 (1923).
[85] *Coppolino v. State*, Fla 223 So.2d 68 (1968).
[86] *Kruse v. State*, Fla 483 So.2d 1383 (1986).
[87] *Daubert v. Merrell Dow Pharmaceuticals, Inc.*, 509 U.S. 579 (1993).
[88] *Flanagan v. State*, 625 So,2d 827 (1993).

ability as dictated by *Frye*."[89] But then, in *Ramirez* (2001)[90]—while still holding that Florida continues to follow *Frye*—the Florida Supreme Court in effect conducted a *Daubert* inquiry into the reliability of proffered knife-mark identification testimony; and concluded that, since it failed this scrutiny, the evidence in question wasn't reliable enough to satisfy *Frye*. (Some legal scholars now describe Florida as a *"Fryebert"* state.)

To accommodate this story formally, Alchourrón and Bulygin would need a rational reconstruction of a whole series of momentary legal orders, each corresponding to a skinny time-sliver of Florida evidence law (i.e., a skinny time-sliver of a segment of the law of one state, itself only a segment of U.S. law more generally). Perhaps it would even be possible, with sufficient ingenuity, to link these time-slivers in an (as yet, so far as I know, hypothetical) tensed deontic logic—or on second thoughts, perhaps we'd need a tensed, paraconsistent, fuzzy deontic logic. From a logician's point of view, this might be an interesting application of formal methods; and such a rational reconstruction *would*, doubtless, be "wonderful to look at in its exactness, symmetry, and formal elegance"—well, but for the fuzzy part, anyway. And it would tell us *something* about real-life legal shifts and changes: something; but certainly not all. It might, for example, help us pinpoint the times at which seat-of-the-pants decisions to deal with specific problems broke with past policies; but it wouldn't help us understand why or how this happened—let alone whether this or that step in the process was in any sense reasonable or rational. As a commentary on this little slice of Florida legal history, Holmes's observation that "[t]he form of continuity has been kept up [. . .] but that form is nothing but the evening dress which the newcomer puts on to make itself presentable by conventional standards"[91] is much more illuminating than any logical model.

Now let me turn to efforts to articulate the logical structure of "arguments by analogy." Such arguments play an important role in attorneys', judges', and legal scholars' reasons for applying, interpreting, and extrapolating a precedent, a rule, etc., in one way rather than another; so one might hope that a rigorous formal treatment that discriminated good from bad (or stronger from weaker) analogical arguments could illuminate some of the issues about legal change and growth that a deontic logic of momentary legal orders cannot.

The legal literature on analogical argument is luxuriant, to say the least—a steamy, tangled jungle in which it would be easy to get hopelessly lost. This time I shall take as my exemplar a well-regarded paper of Scott Brewer's which provides a simple sketch-map we can use to get some

[89] *Brim v. State*, 695 So.2d 268, 271–2 (1997).
[90] *Ramirez v. State*, 810 So.2d 836 (2001).
[91] Holmes 1880, 234.

bearing[92]—a categorization of three families of approaches to (legal) arguments by analogy: "mysticism," which grants such arguments a high degree of rational force, but offers, instead of any account of where that force comes from, only mysterious references to "spiritual acts" (Theodor Heller),[93] "trained, disciplined intuition" (Charles Fried),[94] and the like; "skepticism," according to which legal arguments by analogy have "no definite content or integrity" (Richard Posner);[95] and "Modest-Proposal Rationalism," the view Brewer defends, according to which analogical arguments have some, though modest, rational force—less than the force of deduction or induction—explicable by some identifiable inferential structure.[96]

"The best way to make interpretive sense of what reasoners do when using analogies in arguments," Brewer writes, "is to recognize a regular inferential pattern that guides them in making judgments about the truth of conclusions of analogical arguments on the basis of the truth of their premises."[97] That pattern, he continues, includes elements of abduction, induction, and deduction. He summarizes it like this:

1. Abduction in a context of doubt about the extension of some predicate or the meaning of some text, in which "the reasoner seeks to 'discover' a rule-like sorting of . . . examples." Brewer calls the rule thus "discovered" an "analogy-warranting rule (AWR)."

2. Inductive confirmation or disconfirmation of the AWR to determine whether it is an "acceptable" sorting of the examples. To do this, the reasoner must assess the AWR "against a set of explanatory and justificatory propositions." Brewer calls these "analogy-warranting rationales."

3. Deduction, applying the AWR discovered in the first step and confirmed in the second to the example at issue.[98]

Analogy-warranting rules, Brewer tells us, "state [. . .] the logical relation that obtains between the shared characteristics [. . .] and the inferred characteristics [. . .] ." Analogy-warranting rationales "explain [. . .] why, in the 'eyes of the law' the logical relation among the characteristics articulated by the analogy-warranting rule either does obtain or *should* obtain."[99]

Brewer acknowledges that the idea of abduction comes from Peirce; but is apparently unaware of the analysis of (scientific) arguments by analogy that Peirce had given in 1883—to which, on the surface at least, his analysis of

[92] Brewer 1996.
[93] Heller 1961. My source is Horowitz 1972, 65–8.
[94] Fried 1981, 57, cited by Brewer.
[95] Posner 1993, 1654, cited by Brewer.
[96] Brewer 1996, 951–5.
[97] Brewer 1996, 964.
[98] Brewer 1996, 962–3.
[99] Brewer 1996, 965.

legal arguments by analogy looks rather similar. Classifying arguments by analogy under "probable inferences of mixed character," Peirce writes that such arguments consist of "an induction and an hypothesis [i.e., an abduction] followed by a deduction." For example: "We know that of the major planets the Earth, Mars, Jupiter and Saturn revolve on their axes, and we conclude that the remaining four, Mercury, Venus, Uranus and Neptune, probably do the like."[100] Applying Peirce's analysis to this example:

1. The abduction: an inference from "everything in class X has, for example, properties P_1, P_2, P_3, etc.," and "Q is found to be P_1, P_2, P_3," to "Q is an X." In Peirce's example: Every planet revolves in a circular orbit around a great sun, is nearly spherical, shines with reflected light, is very large, etc.. Mercury, etc., revolve in a circular orbit around a great sun, are nearly spherical, shine with reflected light, are very large, etc.. Hence (abductively) Mercury, etc., are planets.
2. The induction: an inference from "S_1, S_2, S_3 are samples of the X's," and "S_1, S_2, S_3 are found to be R's," to "every X is an R." In the example: Earth, Mars, etc., are planets, and they revolve around their axes. Hence (inductively) every planet revolves around its axis.
3. The deduction: an inference from "Q is an X" and "All Xs are R" to "Q is an R." In the example: Every planet revolves around its axis, and Mercury, etc. are planets. Hence (deductively) Mercury, etc., revolve around their axes.

Despite the superficial similarity, however, there is a very significant difference between Peirce's account and Brewer's: There is nothing in Peirce's account corresponding to Brewer's appeal to analogy-warranting rules or the analogy-warranting rationales that supposedly underpin them. But these in effect require that the shared characteristics on which a legal argument by analogy depends be "*relevant in the eyes of the law*"; and this strongly suggests that arguments by analogy in the law may *differ* quite significantly from arguments by analogy in the sciences.[101]

Scientific arguments by analogy are arguments about how the world is, and their conclusions are factual propositions (such as "Mercury, Venus, Uranus, and Neptune revolve around their axes"). Legal arguments by analogy, however, are not simply about how the world is, but about what decision should be made; and their conclusions—though they are often stated matter-of-factually—are really propositions about what should be done (such as "this case should be treated in the same way that one was,"

[100] In Hartshorne *et al.*, eds., 1931–58, 2.733 (1883).

[101] I reserve judgment as to the correctness of Peirce's treatment of scientific arguments by analogy, which would take me beyond the scope of this paper (as would a scholarly treatment of the shifts in his view of abduction mentioned *supra*, n. 53).

or "this case should be treated as falling under that rule"). When scientists use arguments by analogy, it is as a way of figuring out how the world is; when attorneys and judges use them, their purpose is not to find out whether this case falls under this rule, etc., but to give a persuasive rationale for treating it this way rather than that. Such a rationale indeed requires that the similarities in question be relevant to its being properly so treated. But this means that legal arguments by analogy depend, in a way that scientific arguments by analogy do not, on considerations of policy, of goals to be sought, of values to be served.

I suspect that Brewer has fallen for that false equation of "rational" with "formally-logical," and so fails to recognize the possibility that legal arguments by analogy are rationally assessable though not formally valid. He cites Martin Golding: "The truth or correctness of [the relevance premise] will rest [. . .] on underlying considerations of policy or principle. That is to say, [. . .] on goal-oriented or rights-oriented arguments."[102] This is clearly right; equally clearly, however, it takes us well beyond the reach of formal logic—and yet, not beyond the realm of the reasonable. As I read Charles Fried, precisely this was also (part of) his point: that the persuasiveness of legal arguments by analogy depends on subtle value-judgments, on anticipation of consequences, and on weighing of pros and cons; and that these cannot be reduced to logical formula. As he says, it is "preposterous to imagine that [philosophy] can tell us whether there should be a right to privacy in a public telephone booth or a department store dressing room."[103] In my opinion, it is a mistake to call this "mysticism"; it is plain good sense.

In the last half-century, the "weighing of pros and cons" that seems to me crucial to legal reasoning—as it does to Fried and Golding, and as it did to Holmes—has manifested itself in a line of thought focused on the idea of law as argumentation, dialectics, discourse. Such argumentation, it is acknowledged, will have to appeal to meta-norms, themselves subject to counter-arguments.[104] And, I would add, so on; for the process seems unavoidably open-ended. This is why I am less than sanguine about some of the ambitions of the project that has of late attracted the attention of researchers in Artificial Intelligence and law: to automate the ways in which legal arguments are made, countered, upheld, and defeated.[105] Clearly, computerized models might be a useful tool for, say, an attorney

[102] Golding 1984, 109 (cited in Brewer 1996, 960, n.116).
[103] Fried 1981, 54.
[104] Toulmin 1958; Perelman and Olbrechts-Tyteca 1969; Alexy 1989.
[105] See, e.g., Prakken 1993; Hage 1993; Sartor 1993; Hage 1996, summarizing in part Hage 1997. There is a useful (though apparently un-proof-read!) summary of these developments in Aikenhead 1997. (My summary description in the text is deliberately ambiguous in one respect; it is not altogether clear whether this project is best conceived as descriptive—modelling the reasoning in which judges and attorneys engage—or as normative.)

wanting to check that his client has satisfied all the conditions for making a binding contract; but, ingenious as some of the work in "automated legal reasoning" is, it isn't at all clear to me how such models could supply a (non-arbitrary) ranking of meta-norms,[106] or anticipate the unexpected changes in the world that sometimes prompt legal adaptations.[107]

I have taken only a few examples from a vast array and a wide variety of proposed uses of modern logic in legal theory. Still, they all suggest the same conclusions—the conclusions for which Holmes argued long ago: that the usefulness of formal-logical tools in legal theory is limited; but that this doesn't mean that nothing rational, reasonable, or sensible can be going on, or that an explanation exclusively in terms of the operation of social and economic causes is all that could be hoped for. Here as so often, we must beware of This-or-Nothingism.[108] The life of the law *isn't* logic; but it doesn't follow that judges' decisions can only be arbitrary and capricious.[109]

5. Envoi

Probably, when Langdell observed that law must be a science, or else there could be no justification for teaching law in the universities, he was only expressing the expected boosterist sentiments. (But obviously if he *had* been seriously making this argument, he would, to say the least, have put the cart before the horse.)[110] Equally obviously, I can't embark on a serious discussion of the various (American, European, British, etc.) models of legal education here. But I hope it will be appropriate to close with the thoughts about legal education that Holmes expressed in an 1895 after-dinner speech in honor of Langdell, entitled "Learning and Science."[111] On learning, i.e., legal history and scholarship, Holmes has this to say:

> The law, so far as it depends on learning, is indeed [. . .] the government of the living by the dead. [. . .] The past gives us our vocabulary and fixes the limits of

[106] Henry Prakken comments (Prakken 1993, 4) that which meta-norms are to take priority is an interesting question for further research. As I see it, however, if this were a question with a true-or-false answer, the answer would not be of a kind to be found by formal methods; and, in some instances anyway, it is *not* such a question, but a decision to be made.

[107] For example, that copyright law adequate for the disputes that arise with respect to printed material would become strained to the limit with the technological possibility of electronic file-sharing and such.

[108] I introduced the phrase in Haack 1998, 21, 25; but the phenomenon is ubiquitous.

[109] This was, I believe, precisely the theme of Dewey's commentary on Holmes's view of logic in the law, though he puts it very differently: Holmes's strictures "apply in full force" to syllogistic logic, but don't preclude a place for logic in his, Dewey, much broader, "experimental" sense (which is most akin to the ampler, older usage, "theory of whatever is good in the way of reasoning"). Dewey 1924, 21, 23; see also Dewey 1938.

[110] Moreover, the education of lawyers surely does require some elements of craft, of experience, more easily acquired by apprenticeship than by lectures and seminars.

[111] Holmes (1895), in Marke, ed., 1965, 72–3. All my quotations are from p. 72.

our imagination. [. . .] There is, too, a peculiar logical pleasure in making manifest the continuity between what we are doing and what has been done before.

However, he adds this, on the role of the social sciences:

An ideal system of law should draw its postulates and its legislative justification from science. [. . .] Who here can give reasons [of any kind other than tradition] of believing that half of the criminal law does not do more harm than good? [. . .] How much has reason had to do in deciding how far, if at all, it is expedient for the State to meddle with domestic relations? [. . .] And so I might go throughout the law.

When Holmes writes, as he does here, that "the foundations of the law must be scientific," what he envisages is very different from Langdell's "science of law": His thought is that lawyers and judges should be well-informed not only about their legal system and its history, but also about developments in the social sciences potentially relevant to those questions of consequences, that weighing of pros and cons, in which they will inevitably find themselves engaged.[112] But Holmes's conception is also different from the more imperialist ambitions of the recent law-and-economics "movement": It is more inclusive, insofar as it envisages, for example, a role for psychology in explaining the genesis of crime, and hence in telling us something about the likelihood that rehabilitation is feasible; and at the same time it is more modest,[113] insofar as it acknowledges that what the social sciences tell us about causes and effects can't absolve us from decisions about the ends to be sought. It suggests, in short, a kind of educational eclecticism in which legal scholarship, the social sciences, and logic, are each "something, but not all."[114]

University of Miami, School of Law
Coral Gables, FL 33124-8087,
U.S.A.

[112] Readers of this journal may be interested to know that Holmes describes this conception of the role of the social sciences in the law as one on which "[t]he Italians have recently begun to work." But for U.S. legal scholars Holmes's words will bring to mind the "Brandeis brief": the *genre* named in honor of Louis Brandeis (the first Jewish Justice of the U.S. Supreme Court), who as a lawyer wrote lengthy briefs presenting social-scientific data about the likely effects of decisions and policies, such as his famous brief in the labor-law case *Muller v. Oregon*, 208 U.S. 412 (1908). See Konefsky 1956, 84–92.

[113] Though perhaps somewhat over-optimistic about the prospects for progress in the social sciences, at present often apparently mired in factionalism and political advocacy. See Haack 2003a, 151–77.

[114] My thanks to Mark Migotti, María-José Frápolli, and William Widen for helpful comments and suggestions on various drafts of this paper, and to David Hollander, of the University of Miami Law Library, for his help in tracking down relevant materials.

References

Aikenhead, Michael. 1997. A Discourse on Law and Artificial Intelligence. *Law Technology Journal* 5: 13. Http://www.law.warwick.ac.uk/ltj/5-lc.html.

Alarcón Cabrera, Carlos. 1993. *Normas y Paradojas*. Madrid: Tecnos.

Alchourrón, Carlos E., and Eugenio Bulygin. 1971. *Normative Systems*. Vienna: Springer.

Alexy, Robert, 1989. *A Theory of Legal Argumentation*. Oxford: Clarendon.

Allen, Layman E. 1957. Symbolic Logic: A Razor-Edged Tool for Drafting and Interpreting Legal Documents. *Yale Law Journal* 66: 833–79.

Anderson, Alan Ross, and Nuel D. Belnap. 1975. *Entailment. The Logic of Relevance And Necessity*. Vol. 1. Princeton, NJ: Princeton University Press.

Anderson, Alan Ross, Nuel D. Belnap, and J. Michael Dunn. 1992. *Entailment. The Logic of Relevance and Necessity*. Vol. 2. Princeton, NJ: Princeton University Press.

Anderson, Terence, David Schum, and William Twining. 2005. *Analysis of Evidence*. Cambridge: Cambridge University Press. (1st ed. 1991.)

Arruda, Ayda I. 1980. A Survey of Paraconsistent Logic. In *Mathematical Logic in Latin America*. Ed. A.I. Arruda *et al.*, 1–42. Amsterdam: North Holland.

Atkinson, Brooks, ed. 1940. *The Complete Essays and Other Writings of Ralph Waldo Emerson*. New York, NY: American Library.

Bazhanov, Valentin A. 1990. The Fate of One Forgotten Idea: N.A. Vasiliev and His Imaginary Logic. *Studies in Soviet Thought* 39: 333–41.

Bentham, Jeremy. 1827. Reprinted 1978. *Rationale of Judicial Evidence*. New York, NY: Garland.

Bocheński, Joseph M. 1961. *A History of Formal Logic*. Notre Dame, IN: University of Notre Dame Press.

Boole, George. 1854. Reprinted 1958. *An Investigation of The Laws of Thought, on Which are Founded the Mathematical Theories of Logic and Probability*. New York, NY: Dover.

Brewer, Scott. 1996. Exemplary Reasoning: Semantics, Pragmatics, and the Rational Force of Legal Argument by Analogy. *Harvard Law Review* 109: 923–1028.

Brewer, Scott. 2000. Traversing Holmes's Path Toward a Jurisprudence of Logical Form. In *The Path of the Law and Its Influence: The Legacy of Oliver Wendell Holmes, Jr*. Ed. S.J. Burton, 94–132. Cambridge: Cambridge University Press.

Brouwer, Luitzen E. J. 1952. Historical Background, Principles and Methods of Intuitionism. *South African Journal of Science* 49: 139–46.

Burton, Steven J., ed. 2000. *The Path of the Law and Its Influence: The Legacy of Oliver Wendell Holmes, Jr*. Cambridge: Cambridge University Press.

Bynum, Terrell W., ed. 1972. *Conceptual Notation and Related Articles*. Trans. T. W. Bynum. Oxford: Clarendon.

Carnap, Rudolf. 1950. Reprinted 1962. *Logical Foundations of Probability*. Chicago, IL: University of Chicago Press.

Conte, Amedeo, and Carlos Alarcón Cabrera. 1995. *Deontica de la Validez*. Madrid: Tecnos.

Conte, Amedeo, and Carlos Alarcón Cabrera. 1999. Constitutive Constitutional Reform. *Ratio Juris* 9: 85–93.

da Costa, Newton C.A. 1974. On the Theory of Inconsistent Formal Systems. *Notre Dame Journal of Formal Logic* 15: 497–510.

da Costa, Newton C.A. 1997. *Logiques Classiques et non Classiques*. Paris: Masson.

Da Silva, Virgilio A. 2004. A Fossilised Constitution? *Ratio Juris* 17: 454–73.

Dewey, John. 1924. Logical Method and Law. *The Cornell Law Quarterly* 10: 17–27.

Dewey, John. 1938. *Logic, The Theory of Inquiry*. New York, NY: Holt.

Dickinson, John. 1929. The Law behind Law. *Columbia Law Review* 29: 113–46, 285–319.
Dow, David R. 1993. Gödel and Langdell–A Reply to Brown and Greenberg's Use of Mathematics in Legal Theory. *Hastings Law Journal* 44: 707–44.
Dudman, Victor H. 1976. From Boole to Frege. In *Studies on Frege I: Logic and Philosophy of Mathematics*. Ed. M. Schirn, 109–38. Stuttgart-Bad Cannstatt: Frommann-Holzboog.
Emerson, Ralph Waldo. 1841. Self-Reliance. Reprinted in *The Complete Essays and Other Writings of Ralph Waldo Emerson*. Ed. B. Atkinson, 1940, 145–69. New York, NY: American Library.
Frege, Gottlob. 1879. *Begriffsschrift*. Reprinted and trans. In *Conceptual Notation and Related Articles*. Ed. T. W. Bynum, 1972, 101–203. Oxford: Clarendon.
Frege, Gottlob. 1882. On the Scientific Justification of a Conceptual Notation. Reprinted and trans. In *Conceptual Notation and Related Articles*. Ed. T. W. Bynum, 1972, 83–9. Oxford: Clarendon.
Fried, Charles. 1981. The Artificial Reason of the Law or: What Lawyers Know. *Texas Law Review* 60: 35–58.
Golding, Martin. 1984. *Legal Reasoning*. New York, NY: Knopf.
Grey, Thomas C. 1983. Langdell's Orthodoxy. *University of Pittsburgh Law Review* 45: 1–51.
Grey, Thomas C. 2000. Holmes on the Logic of the Law. In *The Path of the Law and Its Influence: The Legacy of Oliver Wendell Holmes, Jr.* Ed. S. J. Burton, 133–57. Cambridge: Cambridge University Press.
Groernendijk, Jeroen, and Martin Stokhof. 1991. Dynamic Predicate Logic. *Linguistics and Philosophy* 14: 39–100.
Haack, Susan. 1974. *Deviant Logic*. Cambridge: Cambridge University Press.
Haack, Susan. 1978. *Philosophy of Logics*. Cambridge: Cambridge University Press.
Haack, Susan. 1981. Do We Need "Fuzzy Logic"? Reprinted in *Deviant Logic, Fuzzy Logic: Beyond the Formalism*, 1996, 232–42. Chicago, IL: University of Chicago Press.
Haack, Susan. 1993. *Evidence and Inquiry: Towards Reconstruction in Epistemology*. Oxford: Blackwell.
Haack, Susan. 1996. *Deviant Logic, Fuzzy Logic: Beyond the Formalism*. Chicago, IL: University of Chicago Press.
Haack, Susan. 1998. Confessions of an Old-Fashioned Prig. In S. Haack, *Manifesto of A Passionate Moderate: Unfashionable Essays*, 6–30. Chicago, IL: University of Chicago Press.
Haack, Susan. 2003a. *Defending Science—Within Reason: Between Scientism and Cynism*. Amherst, NY: Prometheus.
Haack, Susan. 2003b. Inquiry and Advocacy, Fallibilism and Finality: Culture and Inference in Science and the Law. *Law, Probability and Risk* 2: 205–14.
Haack, Susan. 2004. Coherence, Consistency, Cogency, Congruity, Cohesiveness & Co.: Remain Calm! Don't Go Overboard! *New Literary History* 35: 167–83.
Haack, Susan. 2005a. On Legal Pragmatism: Where Does "The Path of the Law" Lead Us? *The American Journal of Jurisprudence* 50: 71–105.
Haack, Susan. 2005b. Formal Philosophy? A Plea for Pluralism. In *Formal Philosophy*. Ed. J. Symons and V. Hendricks, 77–98. London: Automatic.
Haack, Susan, ed. 2006. *Pragmatism, Old and New: Selected Writings*. Amherst, NY: Prometheus Books.
Hage, Jaap C. 1993. Monological Reason Based Logic. In *Proceedings of the Fourth International Conference on Artificial Intelligence and Law*, 30–9. New York, NY: ACM.

Hage, Jaap C. 1996. A Theory of Legal Reasoning and a Logic to Match. *Artificial Intellligence and Law* 4: 199–273.
Hage, Jaap C. 1997. *Reasoning With Rules*. Dordrecht: Kluwer.
Hartshorne, Charles, Paul Weiss, and Arthur Burks, eds. 1931–58. *Collected Papers of Charles Sanders Peirce*. 8 vols. Cambridge, MA: Harvard University Press.
Heller, Theodor. 1961. *Logik und Axiologie der Analogen Reschtsanwendung*. Berlin: de Gruyter.
Heyting, Arend. 1956. La Conception Intuitioniste de la Logique. *Etudes Philosophiques*, 11: 226–33.
Heyting, Arend. 1966. *Intuitionism: An Introduction*. Amsterdam: North Holland. (1st ed. 1956.)
Holmes, Oliver Wendell. 1880. Book Notice of William Anson, *Principles of the English Law of Contracts*, and Christopher Columbus Langdell, *Selection of Cases on the Law of Contracts*, 2nd ed. *American Law Review* 14: 233–5.
Holmes, Oliver Wendell. 1881. *The Common Law*. Reprinted in *The Collected Works of Justice Holmes: Complete Public Writings and Selected Judicial Opinions of Oliver Wendell Holmes*. Ed. S. M. Novick, 1995, 109–324. Chicago, IL: University of Chicago Press.
Holmes, Oliver Wendell. 1895. Learning and Science. In *The Holmes Reader*. Ed. J. Marke, 1965, 72–3. Dobbs Ferry, NY: Oceana.
Holmes, Oliver Wendell. 1896. The Path of the Law. Reprinted in *The Collected Works of Justice Holmes: Complete Public Writings and Selected Judicial Opinions of Oliver Wendell Holmes*. Ed. S. M. Novick, 1995, 391–406. Chicago, IL: University of Chicago Press.
Horowitz, Joseph. 1972. *Law and Logic*. New York, NY: Springer.
Horwitz, Morton J. 1977. *The Transformation of American Law, 1780–1860*. Cambridge, MA: Harvard University Press.
Horwitz, Morton J. 1992. *The Transformation of American Law, 1870–1960: The Crisis of Legal Orthodoxy*. New York, NY: Oxford University Press.
Howe, Mark DeWolf, ed. 1941. *The Holmes-Pollock Letters*. Cambridge, MA: Harvard University Press.
Hughes, George, and Max Cresswell. 1968. *Introduction to Modal Logic*. London: Methuen.
Kelsen, Hans. 1965. Recht und Logik. *Neues Forum* 12: 421–5, 495–500.
Kelsen, Hans. 1967. Nochmals: Recht und Logik. *Neues Forum* 14: 39–40.
Kelsen, Hans, and Ulrich Klug. 1981. *Rechtsnormen und Logische Analyse*. Vienna: Deuticke.
Kneale, William, and Martha Kneale. 1962. *The Development of Logic*. Oxford: Clarendon.
Konefsky, Samuel J. 1956. *The Legacy of Holmes and Brandeis*. New York, NY: MacMillan.
Kuhn, Thomas S. 1962. *The Structure of Scientific Revolutions*. Chicago, IL: University of Chicago Press.
Langdell, Christopher C. 1871. *A Selection of Cases on the Law of Contracts, with a Summary of the Topics Covered by the Cases*. Boston, MA: Little Brown.
Langdell, Christopher C. 1879. *A Selection of Cases on the Law of Contracts, with a Summary of the Topics Covered by the Cases*. 2nd ed. Boston, MA: Little Brown.
Langdell, Christopher C. 1880. *A Summary of the Law on Contracts*. Boston, MA: Little Brown.
Langdell, Christopher C. 1887. Teaching Law as a Science. *American Law Review* 21: 123–5.
Lempert, Richard. 1988. The New Evidence Scholarship. In *Probability and the Law*

of Evidence: The Uses and Limits of Bayesianism. Ed. P. Tillers and E. D. Green. Dordrecht: Kluwer.
Lewis, Clarence I. 1912. Implication and the Algebra of Logic. *Mind* 21: 522–31.
Lewis, Clarence I. 1918. *A Survey of Symbolic Logic*. Berkeley, CA: University of California Press.
Lewis, Clarence I., and Cooper Harold Langford. 1932. Reprinted 1959. *Symbolic Logic*. New York, NY: Dover.
Łukasiewicz, Jan. 1920. On Three-Valued Logic. Reprinted in *Polish Logic*. Ed. S. McCall, 1967, 16–8. Oxford: Oxford University Press.
Łukasiewicz, Jan. 1930. Philosophical Remarks on Many-Valued Systems of Propositional Logic. Reprinted in *Polish Logic*. Ed. S. McCall, 1967, 40–65. Oxford: Oxford University Press.
Marke, Julius J., ed. 1965. *The Holmes Reader*. Dobbs Ferry, NY: Oceana.
Martin, Richard M. 1958. *Truth and Denotation: A Study in Semantical Theory*. London: Routledge.
McCall, Storrs, ed. 1967. *Polish Logic*. Oxford: Oxford University Press.
Miller, James C. 1962. Two Examples of Syntactic Ambiguities in International Agreements. *Modern Uses of Logic in the Law* 3: 72–7.
Norman, Jean, and Richard Sylvan. 1989. *Directions in Relevant Logic*. Dordrecht: Kluwer.
Novick, Sheldon M., ed. 1995. *The Collected Works of Justice Holmes: Complete Public Writings and Selected Judicial Opinions of Oliver Wendell Holmes*. Chicago, IL: University of Chicago Press.
Oppenheim, Felix. 1944. Outline of a Logical Analysis of Law. *Philosophy of Science* 11: 143–60.
Perelman, Chaïm, and L. Olbrechts-Tyteca. 1969. *The New Rhetoric: A Treatise on Argumentation*, Notre Dame, IN: University of Notre Dame Press.
Pfeiffer, Jon. 1950. Symbolic Logic. *Scientific American* Dec.: 22–4.
Posner, Richard. 1993. Legal Scholarship Today. *Stanford Law Review* 45: 1647–58.
Post, Emil. 1921. Introduction to the General Theory of Elementary Propositions. Reprinted in *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*. Ed. J. van Heijenhoort, 1967, 265–83. Cambridge, MA: Harvard University Press.
Prakken, Henry. 1993. A Logical Framework for Modelling Legal Argument. In *Proceedings of the Fourth International Conference on Artificial Intelligence and Law*, 1–9. New York, NY: ACM.
Prakken, Henry. 2004. Analysing Reasoning about Evidence with Formal Models of Argumentation. *Law, Probability and Risk* 3: 33–49.
Priest, Graham. 1987. *In Contradiction: A Study of the Transconsistent*. Dordrecht: Nijhof.
Priest, Graham, Richard Routley, and Jean Norman. 1989. *Paraconsistent Logic: Essays on the Inconsistent*. Munich: Philosophia.
Prior, Arthur N. 1957. *Time and Modality*. Oxford: Oxford University Press.
Prior, Arthur N. 1967. *Past, Present, and Future*. Oxford: Oxford University Press.
Prior, Arthur N. 1968. *Papers on Time and Tense*. Oxford: Oxford University Press.
Puga, Leila Z., and Newton C. A. da Costa, 1987. Logic with Deontic and Legal Modalities. *Bulletin of the Logic Section of the Polish Academy of Sciences* 2: 71–5.
Puga, Leila Z., Newton C. A. da Costa, and Roberto J. Vernengo. 1992. Normative Logics, Modality, and Law. In *Expert Systems in Law*. Ed. A. Martino, 345–65. Amsterdam: North-Holland.
Putnam, Hilary. 1982. Peirce the Logician. *Historia Mathematica* 9: 290–301.
Rescher, Nicholas. 1969. *Many-Valued Logic*. New York, NY: McGraw Hill.
Ross, Alf. 1968. *Directives and Norms*. London: Routledge.

Routley Richard, Robert K. Meyer, Val Plumwood, and Ross T. Brady. 1982. *Relevant Logics and Their Rivals.* Atascadero, CA: Ridgeview.
Russell, Bertrand, and Alfred North Whitehead. 1910. *Principia Mathematica.* Cambridge: Cambridge University Press.
Sartor, Giovanni. 1993. A Simple Computational Model for Nonmonotonic and Adversarial Legal Reasoning. In *Proceedings of the Fourth International Conference on Artificial Intelligence and Law*, 192–201. New York, NY: ACM.
Stegmüller, Wolfgang. 1976. *The Structure and Dynamic of Theories.* New York, NY: Springer.
Tammelo, Ilmar. 1955. Sketch for a Symbolic Juristic Logic. *Journal of Legal Education* 8: 277–306.
Tammelo, Ilmar. 1969. *Outlines of Modern Legal Logic.* Wiesbaden: Steiner.
Tammelo, Ilmar. 1978. *Modern Logic in the Service of Law.* Vienna: Springer.
Toulmin, Stephen. 1958. *The Uses of Argument.* Cambridge: Cambridge University Press.
Tribe, Lawrence H. 1971. Trial by Mathematics: Precision and Ritual in the Legal Process. *Harvard Law Review* 84: 1329–93.
Tuzet, Giovanni. 2005. Legal Abduction. *Cognitio* 6: 265–84.
Van Eijk, Jan, and Hans Kamp. 1997. Representing Discourse in Context. In *Handbook of Logic and Language.* Eds. J. van Beentham and A. Ter Meulen, 179–237. Cambridge, MA: MIT Press.
van Heijenhoort, Jean, ed. 1967. *From Frege to Gödel: A Source Book in Mathematical Logic,* 1879–1931. Cambridge, MA: Harvard University Press.
Veblen, Thorstein. 1918. Reprinted 1954. *The Higher Learning in America: A Memorandum on the Conduct of Universities by Businessmen.* Stanford, CA: Academic Reprints.
von Wright, Georg H. 1950. Deontic Logic. *Mind* 60: 1–15.
von Wright, Georg H. 1951. *Modal Logic.* Amsterdam: North Holland.
von Wright, Georg H. 1963. *Norms and Action: A Logical Enquiry.* New York, NY: Humanities.
Wang, Hao. 1987. *Reflections on Kurt Gödel.* Cambridge, MA: MIT Press.
White, Morton G. 1949. *Social Thought in America: The Revolt against Formalism.* New York, NY: Viking. (1st ed. 1947.)
Wigmore, John Henry. 1913. Reprinted 1988. *The Principles of Judicial Proof: As Given by Logic, Psychology, and General Experience, and Illustrated in Judicial Trials.* Littleton, CO: Rothman and Co.
Woods, John. 1974. *The Logic of Fiction.* The Hague: Mouton.
Zadeh, Lotfi. 1975. Fuzzy Logic and Approximate Reasoning. *Synthese* 30: 407–25.
Zadeh, Lotfi. 2002. From Computing with Numbers to Computing with Words. In *The Dynamics of Judicial Proof: Computation, Logic, and Common Sense.* Ed. M. MacCrimmon and P. Tillers, 81–118. Heidelberg: Physica.

