A Study After Cardozo: De Cicco v. Schweizer, Noncooperative Games, and Neural Computing

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A Study After Cardozo: *De Cicco v. Schweizer*, Noncooperative Games, and Neural Computing

ROBERT BIRMINGHAM*

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[T]o be judicious is to be weak, and to compare exactly is to be not elect.

Harold Bloom

_They do things better with logarithms._

Benjamin N. Cardozo

I. INTRODUCTION

One finds posturings about logarithms at law by people who do

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* Professor of Law, University of Connecticut. The author thanks Allison Brickley, Sharon Jones, and Louise Van Dyck for help writing this Essay.

_Cf._ ALASDAIR MACINTYRE, AFTER VIRTUE (1984). Maclntyre's title equivocates between 'after' read as 'post' and as 'in pursuit of', suggesting on the one hand that virtue is passé, and on the other that we should strive for it.


2. BENJAMIN N. CARDOZO, _THE PARADOXES OF LEGAL SCIENCE_ 1 (1928).

3. Cardozo wrote in 1928; shift to 1991:

When I first started teaching consumer protection a decade ago, the mathematics of false advertising was simple. If a box or brochure said "100% cotton," you merely took the item in question and subtracted it from the words: any difference was the measure of your legal remedy. Sometimes you had to add in buyer's expertise or multiply the whole by seller's bad faith, but generally the whole reason people even took a class in consumer protection was that you didn't have to learn logarithms.

PATRICIA J. WILLIAMS, _THE ALCHEMY OF RACE AND RIGHTS_ 36 (1991). The dates and Williams' use of the past tense suggest legal logarithms once were esoteric but are okay or inevitable now.
not use them. The language ‘logarithms’ is metaphorical, denoting any strange mathematics.

This Essay studies one great case, De Cicco v. Schweizer, using noncooperative game theory and a neural network. The mathematics is sufficiently strange. The game and network serve different, complementary ends:

Roth recently rashly claims: “Anyone who has ever taken a basic contracts class knows that no common law court in history has ever enforced a gratuitous promise.” Nor does the scholarly literature give a good reason to enforce, or indeed not to enforce, such a promise. Yet a noncooperative game supplies the reason in a class of cases including De Cicco.

A traveller said of a representative cabin in the Texas hill country, it is “one through whose walls a cat could be hurled ‘at random.’” A net provides the equivalent of a cat test for the substantiality of a legal argument, the cat surrogate being thrown through De Cicco without meeting perceptible resistance.

II. KREPS’ GAME

Informally, I first prove a little theorem that a threat cannot succeed, by which below, in Part V, I interpret De Cicco. Its conclusion

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4. I won’t use them either. For a representative use, for example to decide whether the coast of Britain has finite length, see HEINZ-Otto Peitgen et al., Fractals for the Classroom: Introduction to Fractals and Chaos 218-28 (1992). It doesn’t.


6. The “defining characteristic of noncooperative game theory,” which by incorporating rule-governed behavior makes unmistakable the legal relevance of this theory, is its study of “how cooperation (and many other forms of aggregate behavior) can emerge from self-interested individual behavior within a given set of ‘rules.’” David M. Kreps, A Course in Microeconomic Theory 355 (1990).

7. Neural networks are “systems that are deliberately constructed to make use of some of the organizational principles that are felt to be used in the human brain.” James A. Anderson, General Introduction, in Neurocomputing: Foundations of Research xii, xiii (James A. Anderson & Edward Rosenfeld eds., 1988) (hereinafter: Neurocomputing).


2.—A pivotal feature of the net is that it can’t even solve the XOR (exclusive or) problem—can’t learn that of two things, either alone is okay, neither or both bad. A search of the LEXIS Lawrev Library for ‘XOR’ elicits “The following word does not exist . . . .”


10. See infra note 76.

is unintuitive largely because 'threat' in the statement of it deviates from ordinary language. Let A threaten B: undertake to harm B unless B does as A directs. Making good on the threat must harm A as well as B (ordinary language does not make this demand). I assume as well that A's preferences are transparent to B—an assumption that, while sometimes unrealistic, befits my later application.

For example, my dean, Hugh Macgill the First, prefers that I help entertain a guest of the Connecticut Law School, but only if I am not morose: the School comes off poorly then. I prefer solitude to good fellowship, but dislike to mope. If I must dine out, I like to make the best of a bad deal. I tell Hugh: "If you invite me, I will sulk." That is the threat. I hope to forestall his inviting me. Hugh, however, knows my preferences and would not be dean could he not manipulate me through them. He will invite me, and I will be companionable.

I call up a noncooperative game from David Kreps' great graduate text\textsuperscript{12} to demonstrate the theorem. In Figure 1, Hugh chooses at the empty node, I at the filled node. We achieve the parenthetical payoffs, his listed first. By convention play begins at the empty node, hence Hugh chooses first. Restated in this idiom, my threat is to choose \textit{sulk}. If in response he chooses \textit{invite},\textsuperscript{13} we each get 2. The choices make a \textit{Nash equilibrium},\textsuperscript{14} because neither of us can change his choice to get greater utility unilaterally. Were Hugh to choose \textit{invite}, he would have 0 instead of 2. Were I to choose \textit{\normalfont ~sulk}, I would still get 2, because, he not having invited me, I neither sulk nor do not sulk. But \textit{\normalfont ~invite-sulk} is not the sole Nash equilibrium: \textit{invite-\normalfont ~sulk} is a Nash equilibrium too. If I do not sulk Hugh gets 3, his highest payoff; if he invites me, by being companionable I receive 1, not 0.

At \textit{\normalfont ~invite-sulk} the threat succeeds; at \textit{invite-\normalfont ~sulk} it fails. At which Nash equilibrium do Hugh and I find repose? At \textit{invite-\normalfont ~sulk}, because it alone is subgame perfect. Imagine Hugh has chosen to invite me. Now at the restaurant it is my turn to act. The choice before me, at the filled node, is a subgame. A game is subgame perfect if in each subgame each choosing player maximizes her utility. I

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1}
\caption{Kreps' Game}
\end{figure}

\textsuperscript{12} KREPS, supra note 6.
\textsuperscript{13} The tilde ("\textasciitilde") is 'not'.
\textsuperscript{14} See Katz, supra note 8, at 235-41.
receive 1 by not sulking, 0 otherwise. Maximizing my utility, as we all do always, I am pleasant. I choose \( \sim \text{sulk} \), despite my threat. A previous choice to sulk, made before I need act on it, I revoke.\(^{15}\) Hypothetically, my preferences are transparent to Hugh, hence I cannot deter him.\(^{16}\)

15. One objects: pledging to retaliate may alter the circumstances so \( A \) prefers to carry out her threat, one's word counting for something. Also if \( A \) anticipates repeated play, she may prefer to retaliate at some immediate cost \( \text{pour encourager les autres} \). Or \( A \) may become angry, so that hurting \( B \) itself satisfies, offsetting other losses.

The best data here are generated outside contract law, in Hobbes’ state of nature. Inquiry might start at Elmore Leonard, Get Shorty 62 (1990), wherein a character says, “[B]reak his fuckin legs. That’s the first thing they think of, come up with that statement. I say to ‘em, ‘How’s he gonna pay you if he’s in the hospital?’ They don’t think of that.”

It would proceed to an FBI tape of Paul Castellano, perhaps Leonard’s prototype, talking with a capo:

“Look,” said the Godfather, “when we sit down to clip a guy, we have to remember what’s at stake here. There’s some hazard. Guys forget that. They get a guy behind in his vig payments, they get a hard-on about it, right away they wanna whack him. Why? Just because they’re pissed off, they’re aggravated. But what I say is, ‘Hey, you’re making a living with this guy. He gets you aggravated, and right away you wanna use the hammer? How do you get your money then?’”

“It’s means and ends,” Joe Armone said softly. “The idea is to collect. But you know, Paul, I think some guys just take so much pleasure from breaking heads that they’d almost rather not get paid.”

“Yeah, yeah,” the Godfather agreed. “We got some guys like that. Dick-fists, I call ‘em. I’m always sayin’ to ‘em, ‘Just to take a guy out, that ain’t the point.’ Because I’ll tell ya, Piney, anytime I can remember that we knocked guys out, it cost us. It’s like there’s a tax on it or some shit. Somebody gets arrested. Or there’s a fuckup, which means we gotta clip \( \text{another} \) guy, maybe a guy we don’t wanna lose.”


These possibilities do not qualify the game but change it by challenging the stipulated payoffs to \( A \). I am not talking about these other games.

16. Here is an instance of Kreps’ game misplayed by Gauthier, a fine moral philosopher. He proposes this hypothetical case:

B, a university professor in Boston, is offered a position in Dallas. His wife, A, wishes to deter him from accepting the appointment and so tells him that, if he accepts it, she will leave him and remain in Boston, even though she would prefer to accompany him to Dallas. Then if A is indifferent between a lottery that would offer a 70 percent chance that B would stay in Boston and a 30 percent chance that he would go alone to Dallas, and the certainty that both would go to Dallas, .7 is a minimum required probability for deterrent success. . . . A supposes that there is a 50 percent chance that B will accept the appointment in Dallas if she will accompany him, but only a 10 percent chance that he will accept it if she won’t . . . .


One sees at once, as shown below, the game is congruent to that of Figure 1 above with
III. CONTRACT LAW BEFORE DE CICCO

In 1916, New York law unequivocally held that bargained-for

\[ \sim \text{invite} = \text{Boston}, \text{invite} = \text{Dallas}, \text{sulk} = \text{stay}, \text{and } \sim \text{sulk} = \text{go}. \]

So B is choosing at node 1, A at node 2.

Gauthier's Game

Furthermore we can calculate A's (von Neumann-Morgenstern) cardinal utilities by setting A and B staying in Boston = 1 and A staying and B going to Dallas = 0. See JOHN VON NEUMANN & OSKAR MORGENSTERN, THE THEORY OF GAMES AND ECONOMIC BEHAVIOR 15-30 (1953). Then A's utility if both go = .7(1) + .3(0) = .7, the sum of the utilities to A of both staying and B going alone weighted by their probabilities in the lottery. Strictly speaking we know nothing of B's utilities — only what A thinks about them; but we don't need to know.

Gauthier says: "A indeed maximizes her expected utility by her adoption of a deterrent policy, requiring her to form the conditional intention not to accompany B should he accept an appointment in Dallas." GAUTHIER, supra, at 310. To verify this conclusion inspect his claim, quoted above, "A supposes that there is a 50 percent chance that B will accept the appointment in Dallas if she will accompany him, but only a 10 percent chance that he will accept it if she won't . . . ." Id. By it the probability is .5 that A will get what she wants, that B stay in Boston, by his unilateral decision to do so. Hence, forget about that probability; focus on the .5 probability she can affect by her not accompanying him. Doing nothing, she there has utility .5(.7) = .35. Not accompanying him, she gets a lottery with utility .4(1) + .1(0) = .4.

The trouble comes hard on the heels of this conclusion, with what Gauthier thinks his game proves: that it is rational or even possible that A stay if her efforts to deter B's going fail.

I do want to insist that my argument refutes the claim that deterrence is necessarily an irrational policy because carrying out the deterrent intention is not utility maximizing. The argument for the irrationality of deterrence looks only to the costs of deterrent failure. Because there are such costs, it rejects the policy. My argument, on the other hand, relates the probability-weighted costs of deterrent failure to the probability-weighted benefits of deterrent success in order to assess the rationality of forming the conditional, non-maximizing intention which is the core of a deterrent policy. I claim that if it is rational to form this conditional, deterrent intention, then, should deterrence fail and the condition be realized, it is rational to act on it. With appropriate probability weighting, the utility cost of acting on the deterrent intention enters into determining whether it is rational to form the intention. Once this is decided, the cost of acting on the intention does not enter again into determining whether, if deterrence fails, it is rational to act on it. Acting on it is part of a deterrent policy, and if expected utility is maximized by forming the conditional deterrent intention, then deterrence is a rational policy.

Id. at 311-12. From the perspective of Kreps' game, one realizes that Gauthier is wrong two ways. B choosing Boston and A choosing stay is another subgame imperfect Nash equilibrium. B will not be deterred. And A will go.

Compare tort law, intended optimally to deter. The standard of conduct that forms the basis for a finding of negligence is usually determined by a risk-benefit analysis. Judge Learned Hand reduced this formula to: "[T]he probability [may] be called P; the injury L; and the
promises are enforceable unless the promisee is already bound to do what she is bargaining about. The stuff after 'unless' is the preexisting duty rule. This rule is supported by a string citation the court reporter in De Cicco attributed to Bartlett, Wild, and Duvall, attorneys for the defendant. The citation begins, "Pollock on Cont. 161; Vanderbilt v. Schreyer, 91 N. Y. 392; Robinson v. Jewett, 116 N. Y. 40; Kramer v. Kramer, 181 N. Y. 447; 2 Parsons on Cont. 437; Leake on Cont. [6th ed.] 444; . . . ," and extends beyond one's patience. The attorneys cite ordinary judge-made law. Their authorities are cases and texts relying on cases.

Oversimply, there are two broad kinds of artificial intelligence: expert systems and neural networks. Expert systems are relatively frequent in law, neural networks scarce. Expert systems deduce by using rules. Neural networks, as befits the common law, induce from cases. I will build a perceptron, the classic neural network,

burden B; liability depends on whether B is less than L multiplied by P; i.e., whether B [is less than] PL." United States v. Carrol Towing, 159 F.2d 169, 173 (2d Cir. 1947), reh'g denied, 160 F.2d 482 (2d Cir. 1947). Pretend, so there is no problem about duty, that A threatens B, not, "If you go, I'll stay," but, "If you go, I'll kill you." If one just applies the Hand formula to gauge the negligence of the threat, A may be faultless. The harm is B's being killed. Its probability, as Gauthier would have it, is that of B's still deciding to go. The cost of preventing the harm is A's not threatening, which by its loss in expected value to A may exceed the product of the harm to B and its probability. But nobody would think of optimally deterring A from threatening in lieu of optimally deterring her from carrying out the threat. A decides nothing dangerous so early.

17. De Cicco, 221 N.Y. at 431. I shall later refer to this citation as the Bartlett citation. It does not appear at 117 N.E. 807 (N.Y. 1917).


Several properties distinguish classical single-output Rosenblatt perceptrons. (1) They have n input and one output processing elements (PEs), without a hidden layer between. (2) Outputs are 0 or 1. (3) The output of the output PE is an increasing additive function of weighted inputs from the input PEs, hence its output is 0 until the weighted sum exceeds 0, then becomes 1 and stays that. (4) A perceptron learns by adjusting its weights using this function: If its output is and should be 1 or if its output is and should be 0, change nothing. Else, if its output is 0 and should be 1, increase the weights of its inputs that have value 1. Or if its output is one and should be 0, decrease the weights of its inputs that have value 1.

I adjust one part of NeuralWare's perceptron program, see infra note 21. The program's transfer function reports an actual sum if the sum exceeds 0, instead of converting it to 1. See infra note 34. NeuralWare's step function gives this other result, truer to the classic perceptron literature. See John Hertz et al., Introduction to the Theory of Neural Computation 94-97 (1991); H. D. Block, The Perceptron: A Model for Brain Functioning. I, 34 Rev. Mod. Physics 123 (1962), reprinted in Neurocomputing, supra note 7, at 138; Frank Rosenblatt, The Perceptron: A Probabilistic Model for Information Storage and
to model the New York law of preexisting duty preCardozo.

Figure 2 is the preCardozo perceptron. Its boxes are neurons or PEs (processing elements). The lines exhibit the connections between the PEs, which conform to biological axons, synapses, and dendrites. Three PEs comprise the bottom or input layer: a bias term, always 1; a PE registering bargain; and a PE registering duty. The large, hatched boxes indicate outputs of 1; the small, black box indicates an output of 0. Each input PE is connected to the output PE. The perceptron learns by changing the weights on these connections.

One easily teaches a perceptron preexisting duty by exposing it to case law: all combinations of bargain or not and preexisting duty or not, with their resolutions. Figure 3 states the cases for the perceptron in the binary language it understands. Each case is a row in that figure. The cases are coded with '1's and '0's. For example, a '1' in the first column indicates a bargain; a '1' in the second column a duty; and a '0' in that column no duty. Figure 3 instructs that only instances of bargains without preexisting duties produce contracts.

Row 3 of Figure 3, which alone satisfies these conditions for contract, is Hamer v. Sidway (among other cases). That fine, familiar case, like De Cicco, was decided by the New York Court of Appeals. The decedent, William E. Story, Sr., promised his fifteen-year-old nephew, William E. Story, 2d, $5000 not to smoke, chew, drink, swear, or play cards or billiards for money until he was twenty-one. The nephew probably did not promise. The court says he "assented thereto," likely meaning he said he knew an offer was being made. But the nephew performed, and his assignee sued the uncle's executor. The Storys bargained (the entry in row 3, column 1 is '1') and the nephew lacked a preexisting duty (the entry in row 3, column 2 is '0'), the New York Court of Appeals decided. Hence, on the

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21. In this part and elsewhere I use a Compaq 16 Mhz 386 running the perceptron program of NeuralWare, Inc.'s NeuralWorks Professional II/Plus version 4.05.
22. 27 N.E. 256 (N.Y. 1891).
23. Id. at 256.
24. Id.
25. Id.
nephew’s completing performance, the Storys had contracted (the entry in row 3, column 3 is ‘1’).

Row 1 might be *Mills v. Wyman*,\(^{26}\) in which Mills nursed Wyman’s son until he died.\(^{27}\) Wyman promised to pay Mills’ costs,\(^{28}\) and the court would not compel this payment.\(^{29}\) Mills had no duty; nor had he bargained for Wyman’s promise.

Row 4 is any preexisting duty case correctly cited by attorneys for the defendant, Bartlett et al.

Row 2 is most naturally a third-party beneficiary case: a duty exists which the plaintiff did not bargain to impose. Immediately a reader recalls *Lawrence v. Fox*.\(^{30}\) Yet that case won’t work here. Because Lawrence recovered,\(^{31}\) its contract column is ‘1’, and the perceptron cannot learn it, as Part V explains. I substitute a case by Posner denying liability: *Truck Insurance Exchange v. Ashland Oil, Inc.*\(^{32}\) That case held a tort victim an incidental beneficiary of its tortfeasor’s insurance carrier;\(^{33}\) its contract column is ‘0’.

To train the perceptron on the cases of Figure 3, we display them repeatedly. We put the facts 1 or 0 into the bargain and duty PEs. The perceptron decides the cases. Then we tell the perceptron how it should have decided. The perceptron compares its decisions to the correct decisions, then adjusts its weights, moving gradually toward right judgments.\(^{34}\)

In Figure 2 and throughout, large, hatched boxes represent 1s,

\(^{26}\) *3 Pick. 207* (Mass. 1825).

\(^{27}\) *Id.* at 227.

\(^{28}\) *Id.*

\(^{29}\) *Id.* at 212.

\(^{30}\) *20 N.Y. 268* (1859). In *Lawrence*, the court explained that

Holly . . . at the request of the defendant, loaned and advanced to him $300, stating at the time that he owed that sum to the plaintiff for money borrowed of him, and had agreed to pay it to him. . . . The defendant in consideration thereof . . . promised to pay it to the plaintiff . . . .


\(^{31}\) *Lawrence*, 20 N.Y. at 275.

\(^{32}\) 951 F.2d 787 (7th Cir. 1992); *see also* Bankers Trust Co. v. Old Republic Ins. Co., 959 F.2d 677 (7th Cir. 1992) (holding that plaintiff was not bound by settlement between excess insurer and its insured where plaintiff was not party to suit).

\(^{33}\) *Ashland Oil*, 951 F.2d at 789.

\(^{34}\) Here, in some detail, is how the perceptron gets its output from its input. Each input is assigned a weight. Initially the weights are assigned randomly. I draw them from a sample uniformly distributed between −5 and 5. The perceptron multiplies each input, 1 or 0, by its
small, black boxes 0s. Hence the inputs shown for Figure 2 are the facts of a case Bartlett et al.\textsuperscript{35} cited, from Row 4. The perceptron is deciding the case correctly. The inputs besides the bias input are bargain = 1 and preexisting duty = 1; the output is contract = 0.

IV. Reading Cardozo Reading De Cicco\textsuperscript{36}

I need not dwell on the circumstances and resolution of De Cicco, as they belong to our common legal culture. The characters of the case are Joseph Schweizer, the father, in the rag trade in New York; Blanche, the daughter; the Count, with an Italian title but likely little money, whom Blanche loves or covets; De Cicco, a stranger; and Cardozo, a judge. Think respectively of Walter Brennan, Vivian Leigh, Marlon Brando, Fred MacMurray, and Spencer Tracy. Blanche and the Count are engaged. Father promises the Count to pay Blanche $2500 a year. That is in 1902. Blanche and the Count marry; a dec-

weight, then adds the products. If the sum exceeds 0, the perceptron gives output 1. If the sum is 0 or less, it gives 0. The calculations were performed as indicated \textit{supra} note 23.

For instance, let the initial weights be 0.3080 for the bias term; 1.0890 for the bargain term; and 3.4438 for the duty term. Right now, the perceptron knows nothing, so is as likely to get a wrong as a right result. We randomly show it a case, say row 4, which has the inputs 1, 1 (besides the bias input). It multiplies and adds, getting 4.8408. Hence it reports 1—'contract'—which is wrong.

Now it starts to learn, because it sees the right answer, too. It adds, if its answer is 0 instead of 1, or subtracts, if its answer is 1 instead of 0, a set amount from the weight of each input that is 1. I have set this amount at 0.5. In our example the perceptron subtracts 0.5 from each of the three weights.

The weights are now \(-0.1920, 0.5890,\) and 2.9438. We show it Hamer, 1, 0, which it gets right, \(-0.1920 + 0.5890 = 0.3970 > 0.\) If it gets a case right, it leaves its weights alone and proceeds to the next case. That is row 2, with inputs 0, 1, which it gets wrong. Although \(-0.1920 + 2.9438 = 3.232 > 0,\) there is not a contract in this case. Hence it subtracts 0.5 from the bias and duty weights, leaving the bargain weight alone, because the input to that PE was 0.

<table>
<thead>
<tr>
<th>Training</th>
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<tbody>
<tr>
<td>PEs</td>
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<tr>
<td>bias</td>
</tr>
<tr>
<td>bargain</td>
</tr>
<tr>
<td>duty</td>
</tr>
</tbody>
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Presentation and tuition continue, inputs and outputs chosen randomly from the list of Figure 3, until the perceptron gets all four cases right, here at the thirty-seventh sample. The concluding weights are \(-0.6920, 1.0890,\) and \(-1.0562,\) as for training 1 of the figure above. The duty weight changed most, from 3.4438 to \(-1.0562.\) This is as it should be, because the existence of a duty counts against there being a contract. Training the perceptron twice more fills out the figure above. Its weights differ depending on the starting weights and the order in which the cases are presented. For a detailed statement of the perceptron learning rule, see Eduardo D. Sontag & Héctor J. Sussmann, \textit{Back Propagation Separates Where Perceptrons Do}, \textit{4 Neural Networks} 243, 244 (1991).

35. \textit{See supra} note 17.

36. \textit{Cf. Reading De Man Reading} (Lindsay Waters & Wlad Godzich eds., 1989).
ade later De Cicco, an assignee, sues Father on a year's worth of the promise, which Cardozo enforces. 37

What are we to make of the great judge's opinion? The passage that need concern us is:

The defendant knew that a man and a woman were assuming the responsibilities of wedlock in the belief that adequate provision had been made for the woman and for future offspring. He offered this inducement to both while they were free to retract or to delay. That they neither retracted nor delayed is certain. It is not to be expected that they should lay bare all the motives and promptings, some avowed and conscious, others perhaps half-conscious and inarticulate, which swayed their conduct. It is enough that the natural consequence of the defendant's promise was to induce them to put the thought of rescission or delay aside. From that moment, there was no longer a real alternative. There was no longer what philosophers call a "living" option. . . . It will not do to divert the minds of others from a given line of conduct, and then to urge that because of the diversion the opportunity has gone by to say how their minds would otherwise have acted. If the tendency of the promise is to induce them to persevere, reliance and detriment may be inferred from the mere fact of performance. The springs of conduct are subtle and varied. One who meddles with them must not insist upon too nice a measure of proof that the spring which he released was effective to the exclusion of all others. 38

The form of the argument, "It is possible there were these mental events, therefore . . . ," allows nearly any conclusion—possible mental events, unlike unicorns, being lavishly distributed. Empirically, this passage is among Cardozo's silliest. As Cardozo interpreted the record, implausibly, Father bargained against the Count and Blanche, a team, for the title 'Father of the Countess'.

Yet, the record in De Cicco contains nothing to indicate Blanche and the Count ever contemplated rescission or delay. 39 The case is a

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37. De Cicco, 117 N.E. at 811.
38. Id. at 809-10.
39. See supra text accompanying note 38. The individual sentences of the passage elicit these comments: 1. Contract law after Holmes ordinarily does not deal with beliefs (or other mental entities). The preposition 'in' asks to be translated 'because of'. It does not make that
confirming instance of what Judge Posner writing of Cardozo’s opinion in *Palsgraf*\(^{40}\) describes as “Cardozo go[ing] beyond omissions, even misleading ones, and mak[ing] up facts.”\(^{41}\)

There is not an iota of evidence in the record that Father bargained thus. Instead the transcript of the trial suggests he fled the game of Part V, his purchase of the Count being part payment.

I turn to the law. Cardozo distinguished his case from all others.\(^{42}\) He had to distinguish it because the preexisting duty rule, entrenched in New York law,\(^{43}\) precluded, or would have seemed to an ordinary\(^{44}\) judge to preclude, one’s finding a contract. And yet Cardozo wanted to find one. Finding contracts was a propensity of his, seen most starkly in *Wood v. Lucy, Lady Duff-Gordon*.\(^{45}\) For my immediate purposes, de gustibus non est disputandum.

commitment and retreats to ‘while having’. 2. Here ‘offered’ must be a term of ordinary language, lest Cardozo commit a petitio. But it gets its force from its technical legal use. Nothing in the record indicates the promise was made or received as an inducement, or made both to Blanche and to the Count. 3. They did marry. So that is true at least. The record neither discloses nor denies delays. 4. Fine, we do not know why they acted as they did. 5. To put a thought aside, you first must entertain it. Nothing in the record supports a claim that either Blanche or the Count contemplated rescission or delay. 6. & 7. These alone seem straight out false, unless merely metaphysical. Neither ‘real’ nor ‘living’ may make an empirical claim. I do not know the philosophers Cardozo had in mind. 8. Again, the record does not reveal that any minds were diverted. Otherwise, Posner puts the point, not a bad one, this way: “The proof of a counterfactual (what would have happened if . . . ) is often difficult, and, within reason, doubts are resolved against the wrongdoer.” Sunds Defibrator AB v. Beloit Corp., 930 F.2d 564, 567 (7th Cir. 1991). But Cardozo stretched to assign wrongdoing. 9. The need for inducement is wholly hypothetical. The stance here is to accept as true what is merely possible. 10. Sure. 11. Cardozo’s rejection of “too nice a measure of proof” rings hollow in a construct devoid of proof at all.

\(^{40}\) *Palsgraf* v. Long Island R.R., 162 N.E. 99 (1928).
\(^{42}\) See DeCicco, 117 N.E. at 808-09.
\(^{43}\) See *id.* at 808. Cardozo notes in passing that “[t]he Courts of this state are committed to the view that a promise by A. to B. to induce him not to break his contract with C. is void.” *Id.*
\(^{44}\) “It is strange to talk of Hercules when your starting point is Harry Blackmun.” JOHN T. NOONAN, JR., *PERSONS AND MASKS OF THE LAW* 174 (1976). Cardozo is Hercules near enough.
\(^{45}\) 118 N.E. 214 (1918) (“A promise may be lacking and yet the whole writing may be ‘instinct with obligation’ . . . .” *Id.* at 214.).
Here is Cardozo's distinction: if a promise runs jointly to parties bound by a contract between them to do what the promisor is bargaining for, their not rescinding their contract, as they might do, is consideration for the promise.\footnote{See \textit{De Cicco}, 117 N.E. at 808-09.}

Cardozo decided, and the passage we are investigating presupposes, that Father's promise, although in form directed to the Count only, ran also to Blanche. Or the passage I quote implicitly argues for this proposition. It is difficult to describe duplicity in the language of ordinary scholarship.

Let me relate the legal rule to the computations of Part III. Cardozo is telling us, in terms of perceptrons, that Figure 2 misspecifies the network. The true perceptron, that of Figure 4, has a fourth input PE, in addition to bias, bargain, and duty PEs. This fourth PE is sensitive to the promise running jointly to parties who have contracted, as Cardozo says the promise does in \textit{De Cicco}.

The true data then are those of Figure 5. The first difference from the data of Figure 3, upon which contract law appeared to be built before Cardozo, is that the new data include a column, labelled 'joint', supplying the new input. The entry in that column is '1' if a promise runs jointly to parties bound and '0' if not. For the first four rows, which correspond to those of Figure 3, this input is '0', signifying that in the Bartlett citation\footnote{See supra note 17 and accompanying text.} promises did not run to contracting parties jointly. The second difference between the two figures is that the fifth row of Figure 5 — 1, 1, 1, 1 — is new. Of all the cases that had come before the Court of Appeals only \textit{De Cicco} goes in that row.

Because he faced a case of first impression, Cardozo could freely select '1' or '0' in the \textit{contract} column of the fifth row of Figure 5; he chose '1'. The subterfuge of adding a variable to ratify a result succeeds everywhere except in quantum mechanics.\footnote{See generally John S. Bell, \textit{Speakable and Unspeakable in Quantum Mechanics} (1987).} One imagines Cardozo in the enriched context speaking thus: "The trouble is that the correct perceptron, from Figure 4, was trained on the wrong data:

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{Inputs} & \textbf{Output} \\
\hline
\text{bargain} & \text{duty} & \text{joint} & \text{contract} \\
\hline
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 \\
\hline
\end{tabular}
\caption{True Data Set for Preexisting Duty}
\end{table}
A STUDY AFTER CARDOZO

those of the first four rows only of Figure 5. It should have been trained, and should now be trained, on all rows."

For rows 1 through 4 of Figure 5, the input for \textit{joint} is uniformly 0. Hence the PE recognizing jointness is redundant. A suitable program designed to prune a network would remove it.\textsuperscript{49} A perceptron trained on the first four rows of Figure 5, without \textit{De Cicco}, may decide that case either way. Initially, the weight for the input \textit{joint} is randomly assigned. Training does not alter it because its multiplicand, the input itself, is always 0. Figure 4 above shows the perceptron deciding \textit{De Cicco} correctly.\textsuperscript{50}

Anne Gardner’s influential book advocating legal expert systems, which are rivals of neural networks in artificial intelligence, says that ideally,

an AI program would be able to be told about new sources—newly decided cases, for instance, or old cases not included in the initial knowledge base—and to fit these cases into its existing rule structure, perhaps forcibly. The decisions, used as annotations to the rules, could begin to change the rules’ status and their meaning.\textsuperscript{51}

The perceptron is acting out Gardner’s ideal scenario with these correspondences:


cases \rightarrow \text{rows of Figure 5}

newly decided case \rightarrow \text{row 5}

rules \rightarrow \text{connection weights.}

\textsuperscript{49} See Jocelyn Sietsma & Robert J.F. Dow, \textit{Creating Artificial Neural Networks That Generalize}, 4 NEURAL NETWORKS 67, 70 (1991) (telling how to remove superfluous PEs, superfluous usually because their weights after training approach 0).

\textsuperscript{50} The first column of the following table shows weights that decide the case as Cardozo did. The second column shows weights that decide the case differently. That is, $-0.0206 + 0.4533 + (-0.9986) + 4.9682 = 4.4023 > 0$, while $-0.2324 + 0.5998 + (-4.5750) + (-3.4445) = -7.6521 < 0$. These calculations use the weights 4.9682 and $-3.4445$ at \textit{joint}.

\textsuperscript{51} VON DER LIETH GARDNER, \textit{supra} note 18, at 30.
One may say a perceptron, unlike an expert system, induces its own rules. To speak thus is unperspicuous, we will see at once. As Gardner wants, a perceptron accommodates new cases by modifying these rules. Moreover, a perceptron exceeds Gardner's fondest imaginings. Gardner announces things like "each opinion can be viewed as a subset of precisely the rules that are stored in the knowledge base." She reifies rules and subordinates cases to them. That is not the perceptron's way. For a perceptron, the cases are real, the rules epiphenomenal. To seek rules in a perceptron is like searching a person for a soul. Rules are theoretical entities one posits to explain perceptron behavior.

V. UNDERSTANDING DE CICCO

Here is the flat state of the art in reading De Cicco:

The defendant in De Cicco v. Schweizer plainly lived to regret his promise of annuity to his daughter Blanche. The suit on the promise was brought by an assignee—suggesting, perhaps, that Blanche's marriage to Count Oberto has turned out no better than we expected. Yet however severe the father's regrets at this stage, and however reluctantly we require him to continue the annual payments to a faceless speculator, the case offers one of the easiest illustrations of the proposition that the enforceability of gratuitous promises must be regarded as a benefit, not a detriment, to those persons who make them. If Joseph Schweizer had not been able to make a legally binding promise to his daughter, he would at best have had to incur significant additional expense to achieve the same result in another way, such as by creating a trust to make the payments. The alternative, in all likelihood, was to give up the chance to see Blanche married to the Count. At a minimum, the ability to make an enforceable promise allowed Schweizer to achieve his ends at lower cost; and it may well have afforded the only means of achieving them.

But the law is not so bland; starting from zero. The forthright interpretation of the facts of De Cicco is that Blanche, pointing to the Count, said, "Daddy, buy me that," and he did. By this reading, the engagement is peripheral, and the preexisting duty rule ought not

52. Id. at 49.
53. Or our judges' or our own behavior.
54. Andrew Kull, Reconsidering Gratuitous Promises, 21 J. LEGAL STUD. 39, 62 (1992) (footnotes omitted); see infra note 76.
55. Father got what Blanche wanted at the low end of a reasonable price in 1902 dollars. "Going prices" for adoption or brief marriage conferring title are "from $100,000 for a common count, to $1 million for a proper prince." Philip Revzin, Want to Be Called Prince or Countess? Here Is the Deal, WALL ST. J., Feb. 27, 1989, at A1.
interfere with the purchase. Yet the reading is anachronistic; we feel less insistently today than in 1902 the contractual weight of an engagement—now the woman gets and keeps a ring. Decided thus, as just a routine case of bargained-for exchange, De Cicco wants greatness.

Cardozo decided he would speak interestingly rather than truly. There is something to be said for this stance, and I will say it, keeping in mind the constraint that a judge ought not decide a case that is too hypothetical. The reason the judicial system is subsidized, putting the best face on things, is that a judge not only settles disputes between parties to a case, but builds law that will guide subsequent parties and courts similarly situated. The claim presumes positive externalities from adjudication. The size of these externalities depends on what kind of case the judge is deciding. De Cicco generates more externalities as a family romance than as an installment sale. Nor does contemplating the case as enforcement of a simple marriage settlement, as is often done, stir the blood. The finding of a family romance coheres with the forthright reading that the engagement matters not a whit. By this reading though not Blanche but the Count is superfluous. Father and Blanche have the big relation. The Count is John Ireland in the fight between John Wayne and Montgomery Clift in Red River.

Hence I must address the father-daughter relation. The record in De Cicco, which Cardozo respects little, helps here. Of course, the precedent to start with is King Lear, illustrating poor estate planning. Father, unlike Old Lear, prudently didn't relinquish all assets at once. But why give anything to Blanche ever? For love, defined with an economist's close calculation of profit and loss. A and B love each other if they "have interdependent utility functions." In De Cicco the dependence is only Father's utility on Blanche's because nothing in the record indicates Blanche cares a fig for Father.

By way of illustration, pretend I have a friend, call her Clytemnestra. She buys a washer and dryer combination from Sears. The bank dishonors her check for insufficient funds. Clytemnestra knew the bank would do that. As a fugitive, she telephones me, asking, "Pay Sears lest I go to jail." I love Clytemnestra, so I have a big opportunity. At a price of $500, I can vastly increase her happiness,

57. See supra text accompanying note 41.
58. See William Shakespeare, King Lear, act 1, sc. 1.
60. Only partly true.
hence somewhat increase my own happiness. The size of my gain depends on the strength of my love. Because the opportunity cost of $500 is less than my expected gain, I pay, thus releasing Clytemnestra to shop again. It is her good luck that my preferences run this way. She is maximizing her utility by incurring the optimal (for her) amount of trouble: Less trouble and she could extract less money from me, I lacking the opportunity to make her so happy. More trouble costs her more than it is worth. The game ends when the bank dishonors her check for $50,000, given as down payment on a farm. I decline to pay, preferring to buy, say, a SUN 3/160C.

The trial court awarded De Cicco eighty-four dollars. That is distant from the $2500 he sought. The trial court thought Father proved that he had piecemeal advanced Blanche $2416. Inspect the record. Typically, Father testified, "Q. You mean to tell us that you just handed her the $80? A. I just handed her the $80," and "Q. Did you send her the pearls? A. I sent her the pearls." He said much more. "A. The Countess came to New York alone; she had no money, and she wanted some," Father described. "I gave it to her, and she told me to advance some money on account of her allowance. I told her there was nothing due on her allowance, that I paid her allowance; she said, Well, take it off from the allowance that comes due next year." Again, "A. . . . She wanted to separate from her husband, and she asked me for the money," to which plaintiff successfully objected: "Mr. Schneiderman: I object to that, and ask that it be stricken out, and ask your Honor to instruct the jury to refrain from considering it. The Court: Yes, strike it out." All this, abstracting from nobility and the difference between pearls and washer-dryers, is just Clytemnestra telephoning me about Sears.

Let us return to the game at hand. Figure 6 states the game abstractly, prior to Father's promising the Count. Blanche chooses ~spend or spend at the unfilled node. By ~spend, I mean that she frugally apportions her disbursements over time, so when she comes

62. Record at 1.
63. Record at 37.
64. Record at 48.
65. The record is enlivened only by a witness, expert in Italian, sounding presciently like Stanley Fish talking about textual communities. See Record at 22-26; STANLEY FISH, DOING WHAT COMES NATURALLY: CHANGE, RHETORIC, AND THE PRACTICE OF THEORY IN LITERARY AND LEGAL STUDIES (1989).
66. Record at 49.
67. Id.
to New York alone, she is not destitute. By spend, I mean that she promptly dispenses the money Father gives her, and pleads to him poverty and distress, intending to extract new money.

The unintuitive aspect of this game, the part that is hard to keep in focus, is Father always chooses conditionally and twice: once give or ~give if Blanche chooses spend, a second time give or ~give if Blanche chooses ~spend. That is why there are two filled nodes in Figure 6. I state parenthetically the possible payoffs, Blanche’s first. The numerals read ordinally tell only how Blanche and Father rank the payoffs. For instance Blanche likes spend-~give . . . least, because she gets −2. Father likes ~spend- . . . ~give best, because he gets 2.

So, we think, Blanche, like Clytemnestra, will spend, while Father, as I did, will give at his top node, with baseline payoffs 0 for each. Father is better off, if Blanche chooses spend, to give, receiving 0 instead of −1. So of course is Blanche better off, 0 to −2. The pair of payoffs (0,0) is Pareto superior 68 to (−2,−1), obtained if Father fails to give at the top node. More completely, spend-give-~give, stating Father’s choices top first, is a subgame perfect Nash equilibrium. 69 That is, Blanche is better off spending if otherwise Father will not give than not spending. What Father chooses at his bottom node does not affect his payoff, given Blanche chooses to spend, because the play proceeds through the top node.

But both outcomes with Blanche choosing ~spend are Pareto optimal. Elsewhere someone is worse off. Especially, (1,1) dominates (0,0). Blanche and Father’s problem is how to get to (1,1). But first consider why the payoffs are 1 each if Blanche does not spend yet Father gives.

We return to Clytemnestra, who often wasted her mornings elud-
That Blanche or Clytemnestra not spend and that concomitantly we give is neither Father's first choice nor mine. In this regard the new game replays Kreps' game. Better for me that Clytemnestra buy nothing, that she avoid Sears. So I threaten her: "I shall not pay." My threat of course as we saw fails.\textsuperscript{70} Comparably Father states his choices \textasciitilde{give} and \textasciitilde{give} at the two filled nodes, hoping then Blanche will choose \textasciitilde{spend}, getting $-1$ instead of $-2$ if she spent. Their choices, with payoffs $(-1, 2)$, are Pareto optimal, because elsewhere Father does worse. And the choices are a Nash equilibrium. As we have just seen, Blanche by spending benefits less, assuming Father will not give in any case. Remember, Father has chosen \textasciitilde{give} at both filled nodes. Assuming Blanche does not spend, his decision at the top filled node doesn't affect his payoff because the play now passes through the bottom node. Father's utility from not giving at the bottom node surpasses that from his giving there 2 to 1.

Yet we know Blanche will spend nevertheless. Her spending positions Father at his top node, where his resolution not to give falters, because by giving he receives 0, not $-1$. This is the old Schelling game of whoever can commit herself first wins.\textsuperscript{71} The choices \textasciitilde{spend}, \textasciitilde{give}, \textasciitilde{give} are a Nash equilibrium. They are not subgame perfect, however, because they require that Father choose against his interest. We know that $(1, 1)$ dominates $(0, 0)$: Blanche and Father \textit{both} have more utility there than at the subgame perfect Nash equilibrium. That's the rub. So Father promises Blanche, "Don't spend, I'll give anyway." Imagine—counterfactually it will turn out—that Blanche does not spend. Then Father is choosing at the bottom filled node of Figure 6, and, maximizing his satisfaction, chooses \textasciitilde{give}. This is straight Hobbes not Schelling: "[H]e which performeth first, doth but betray himself to his enemy . . . ."\textsuperscript{72} Blanche, no slouch she, can predict this choice, hence spends despite Father's promise. So Blanche and Father, unaided, are stalled at the Pareto inferior $(0, 0)$. Enter the law, Hercules, to get \textit{De Cicco almost} right. Cardozo

\textsuperscript{70} See supra text accompanying notes 13-16.

\textsuperscript{71} See generally THOMAS C. SCHELLING, THE STRATEGY OF CONFLICT (1960).

\textsuperscript{72} THOMAS HOBBES, LEVIATHAN 90 (Michael Oakeshott ed., 1960).
enforces Father’s promise give at the bottom filled node. Blanche then can safely choose ~spend. The parties achieve a Pareto optimum. It is not a Nash equilibrium. So what: it is Pareto superior to the one that is subgame perfect.

Simply enforcing donative promises may trump two other efforts toward this optimal result. In a delicate familial context, the bargaining that provides consideration is tacit and unlikely to be uncovered by a court. Likewise, although Blanche might have relied on Father’s promise, reliance on a gratuitous promise in every case I know about consists of spending—Johnny buys a car and so forth—~not of refraining from spending as required here.

Still, Cardozo should have ruled for Father. In the end he erred. The true rule is: enforce the promise unless the promisee assigns it. Enforcement of an assigned promise puts the parties back at the Pareto inferior (0,0) as follows. Blanche, by assigning, has spent in this wise. Father promised to pay her $2500 a year. If she can sell the annuity, she can exhaust the proceeds and come again to him for money, pleading her endemic distress. Thus Father lacks incentive to promise in the first place. The court’s enforcement of an assigned promise cancels the advantage that enforcing the promise unassigned achieves.

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73. See Appendix, 4 A.L.I. Proc. 98-103 (1926) (discussing possible outcomes where the promisee, here Johnny, spends less than the promisor promised).
74. De Cicco, 117 N.E. at 808.
75. Id.
76. The literature otherwise shows not much reason either to enforce or not to enforce gratuitous (donative) promises.
1. Reasons Against Enforcement. First, because our legal system is subsidized, parties arguably litigate too much. To restrict legal redress other than by price, contract law, the reasoning goes, should refuse to enforce small donative promises, whose costs of enforcement exceed the attendant social gain. See Richard A. Posner, Gratuitous Promises in Economics and Law, 6 J. LEGAL STUD. 411, 412-13 (1977). Second, a donative promise is more likely to be imprudently made than a bargained-for promise. Yet a donative promise’s formality signals prudence. Melvin Aron Eisenberg, Donative Promises, 47 U. CHI. L. REV. 1, 12-13 (1979). Then contract law might profitably enforce big, attentively expressed donative promises.
2. Reasons for Enforcement. The literature makes out only a weak case for enforcement of donative promises. That case is on two levels. At the first level, letting an agent do what she wants, in this instance bind herself, increases social wealth. An informed agent contracts only if that lets her be better off. It is likely, and in any case can be assured by complementary legal rules, that her behavior creates no external diseconomy. Hence, by making herself better off, she pro tanto makes the world better off. The language here is that of Pareto superiority. At the second level, a promisor needs a reason to prefer to contract. The cost of a donative promise to the promisor is the utility of the discounted amount promised. Her benefit is the promise’s expected value to the promisee—the utility of the sum promised multiplied by the subjective probability on the part of the promisee that the promisor will perform. Posner, supra, at 412-13. Law standing ready to enforce the promise increases that probability by
VI. CAT TEST

I invert the misogynist Samuelson's Razor, "All economic regularities that have no common-sense core that you can explain to your wife will soon fail," to get the cat test for insubstantiality of a legal theory. That is, "Any principle you can explain to your perceptron is not profound." A perceptron cannot handle variables that interact nonlinearly, in the sense it can compute \( y = ax + b \) but not, say, \( y = x^2 + c \). A theory is uninteresting unless its variables interact.

My target is the likes of Farnsworth and Young. They compliment Cardozo's "dazzling ingenuity" in De Cicco and call the case "A Judicial Tour De Force." It is nothing of the kind. Cardozo has "depth as a brook is deep when one cannot see the bottom."

Consider the career of the perceptron. Rosenblatt's 1958 paper inventing this net "created a sensation."

In the popular history of neural networks, first came the classical period of the perceptron, when it seemed as if neural networks could do anything. A hundred algorithms bloomed, a hundred schools of learning machines contended. Then came the onset of the dark ages, where, suddenly research on neural networks was unloved, unwanted, and, most importantly, unfunded. Communicating to the promisee the promisor's intention to keep it — or at least the promisee thinks she will get the money whatever the promisor intends. Hence the promisor may retain the same benefit while reducing the amount she promises, thereby lessening her cost. Her gain is straight social gain because she eliminates rather than shifts the cost. This is the tree Kull is barking up in Kull, supra note 54, at 62.

The analysis is dubious because if the promisor does pay, the promisee later gets more utility than she now expects. Or conversely, pretend the promisor never intends to pay. Without contract law her cost is nothing but transaction costs, while with contract law it is again the sum promised. The expected value to the promisee does not fall as far as the expected cost to the promisor, because the promisor knows she will not pay, although the promisee may only doubt she will pay.


78. See supra text accompanying note 11.


81. See Rosenblatt, supra note 20.

82. Introduction to Rosenblatt, supra note 20.

The book *Perceptrons* likely caused AI to abandon this technology; at least it focused disaffection. Rosenblatt may have done the honorable thing. What matters for us is the source of shortcoming. Notably the book "concluded that since a perceptron could not perform an exclusive OR operation, it was so limited as to be uninteresting." The exclusive OR operation (XOR) ascertains whether one or the other of two conditions is satisfied, although not both. For example, John W. Davis is said to have argued before the Supreme Court in successive weeks on alternate sides of a single issue.

*Justice X:* Weren't you here only last week asserting exactly the opposite proposition?

*Davis:* Yes, your Honor.

*Justice X:* What have you to say for yourself?

*Davis:* I just hope I don't lose both cases.

Davis was appealing to XOR.

84. MINSKY & PAPERT, supra note 20, at 12.


87. Or so the story goes. I can't find it in WILLIAM HENRY HARBAUGH, LAWYER'S LAWYER: THE LIFE OF JOHN W. DAVIS (1973). The 45 'Davis' references in the LEXIS Lawrev Library either speak unenthusiastically of his representing South Carolina in a companion case to Brown v. Board of Education, 347 U.S. 483 (1954), or quote his noble and self-serving statement that having J. P. Morgan as a client did not corrupt him.
Perceptrons fail at XOR because the problem is not linearly separable. XOR would be linearly separable if one could draw a straight line\(^8\) that divides the cases decided differently. A common index of the difficulty of a problem is the degree to which it lacks linear separability.\(^9\) By this index, perceptrons solve only easy problems.

Figure 7 shows the linear separability of the preCardozo preexisting duty rule. Plot bargain and duty on the \(x\) and \(y\) axes. At \((1,1)\) there are both, at \((0,0)\) neither. The filled circle shows contract, the empty circles no contract. The perceptron implicitly represents the preexisting duty rule by a line like duty \(=\) bargain - 0.5 in Figure 7. This line separates \((1,0)\) contract from the other cases of no contract. The preexisting duty rule is an AND problem—two conditions met at once—that perceptrons succeed at.

\(^{8}\) Or more generally a hyperplane. See Sontag & Sussmann, supra note 34, at 244.

\(^{9}\) See P. Gallinari et al., On the Relations Between Discriminant Analysis and Multilayer Perceptrons, 4 NEURAL NETWORKS 349, 352-53 (1991) (presenting a set of experiments on problems with increasing degrees of nonlinearity). Of course the predicate 'easy' has a big intuitive component here. What is easy to Feynman may be hard for Fido and vice versa.

A few technical remarks:
1. "The parity problem (or XOR, its \(N = 2\) version) is often used for testing or evaluating network designs. It should be realized, however, that it is a very hard problem . . . ." HERTZ, supra note 20, at 131.

2. The measure of linear separability is the largest eigen value of the matrix that is the outer product of the total dispersion matrix and the dispersion matrix between classes. A value 0.0 is total inseparability; 1.0, complete separability. The linear separability of XOR (parity) is 0.002. Gallinari, supra, at 352-53.

3. The minimum distance of a case from the plane perpendicular to the weight vector of the optimal perceptron for a problem measures how hard this problem is for perceptrons. A negative distance indicates no perceptron can solve it. In Figure 7, this measure speaks to the breadth of the channel between 0,0 and 1,1, on one hand, and 1,0 on the other. The difficulty for AND is \(1/\sqrt{17}\), for XOR, \(-1/\sqrt{3}\). HERTZ, supra note 20, at 99-100.
I give three examples of cases perceptrons cannot reconcile by reason of the "ever present" XOR problem.\textsuperscript{90}

1. Illusory Promise. If a party stands mute, she does not contract, absent a few exceptions like prior dealings. If a party says, "I promise $x$," or says, "I promise not-$x$," she contracts, presupposing consideration to support the promise. If she says "I promise $x$ or not-$x$," again she does not contract.

2. Rape. An anomaly in the law of rape for a long time was its exonerating a rapist if his victim conceived, because conception implied consent.\textsuperscript{92} But earlier, before Gratian's rule, AD 375-383, there was little rape law at all. Canon law "did not study, comment on, or codify simple rape."\textsuperscript{93} Then assault (0,1 in Figure 8), and fornication (1,0) were offenses. Failing to assault or fornicate (0,0) was by default licit. Simultaneously assaulting and fornicating (1,1) was too. "Why these two notions [assault and fornication] were not brought together before the time of Gratian would be worthy of further study."\textsuperscript{94} Plot sex and violence along the axes of Figure 9, whose filled circles show culpability and empty circles, legal innocence. So at (0,1), nonviolent sex, the accused is culpable; at (0,0), inaction, he is innocent.

3. Third-Party Beneficiary. Return to Figure 3 representing contract law preCardozo. I said Lawrence v. Fox\textsuperscript{95} would not fit in Row 2.\textsuperscript{96} Were we to put it there, in place of Posner's cases,\textsuperscript{97} we would replace 0, 1, 0 with 0, 1, 1 and get the matrix of Figure 8. This is an XOR a perceptron can't resolve: déjà vu all over again.\textsuperscript{98} Geometri-
cally, the cases would fit Figure 9, not Figure 7.

The poverty of perceptron thinking in hand, reflect on Cardozo’s strategy to decide De Cicco. 1. Find a fact in De Cicco but not in the Bartlett citation. Cardozo found Father’s promise running to the mutually bound parties jointly. He did not so much find this fact as create it. This was unnecessary because there will always be such a fact—else De Cicco is res judicata. 2. Let the case turn on this revealed or invented fact. 3. Decide the case as you desire. The cat test disqualifies 1-3 as not profound.

In perceptron terms, Cardozo need only pick a big positive weight, perhaps 25, for joint in Figure 5. A perceptron trained on the preexisting duty cases before De Cicco, that is, on the first four rows of Figure 5 only, then finds contract = 1. Conversely picking a big negative weight forces a decision contract = 0.

99. See supra note 17 and accompanying text.
100. See supra notes 39-41 and accompanying text.
101. Still, the cat test is no panacea.

You can give any conclusion a logical form. You always can imply a condition in a contract. But why do you imply it? It is because of some belief as to the practice of the community or of a class, or because of some opinion as to policy, or, in short, because of some attitude of yours upon a matter not capable of exact quantitative measurement, and therefore not capable of founding exact logical conclusions.

Oliver Wendell Holmes, The Path of the Law, 10 HARV. L. REV. 457, 466 (1897). The point recast to my context is you can always add a new variable, but it makes a difference, which a perceptron can’t pick up, why you added it. Cardozo adds the variable just to get the result. He has no functional reason, and gives none, to separate joint rescission (De Cicco read tenden- tiously) from unilaterally abstaining from seeking rescission (Vanderbilt etc.) as sources of consideration.

The cat test is to some extent a creature of form. To see this, consolidate the cases of Figure 9 by adding a variable with value 0 for rows 1 and 4 and with value 1 for rows 2 and 3 of Figure 8. A perceptron can learn this distinction. We get a reduction, though, since a new column for the variable would be just the output column of Figure 8. All XOR (everything) is lost by this consolidation. Now, focus on the highly interactive game of Part V, opaque to the perceptron. A similar loss results from a perceptron-friendly version of it.

102. The constraint here is a distant one. Let us assume that a case came up with no bargain, no duty, but a promise to parties jointly. The perceptron, if the weight at joint is very much larger than the other weights, discovers a contract in this case too. The new case, call it ‘case ‘X’, is a sixth row in Figure 5, which, if the law ought not enforce gratuitous promises, reads ‘0, 0, 1, 0’. The perceptron gets this case wrong.

<table>
<thead>
<tr>
<th>Training</th>
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<tbody>
<tr>
<td>PEs</td>
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<tr>
<td>bias</td>
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<td>bargain</td>
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<td>duty</td>
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<tr>
<td>joint</td>
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But the six cases together remain linearly separable. Therefore one can retrain the perceptron already trained on the data of Figure 5 to decide all successfully. The table shows two
VII. CONCLUSION

In this Essay, a noncooperative game does constructive, a perceptron evaluative duty. The game gives a general reason to enforce gratuitous promises. The perceptron provides a criterion for the substantiality of legal argument. Cardozo is still elect, in the language of the Bloom epigraph. But then God is Calvinist and election an act of grace.

retrainings, starting from joint = 25. Basically, the weight for the joint PE falls and that for the bargain PE rises until the latter exceeds the former.

103. See supra note 1.